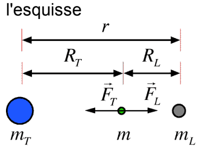
Solutions: gravitation

1 donné: $r = 3{,}9 \cdot 10^{8}\ m\,;\ m_L = 7{,}3 \cdot 10^{22}\ kg\,;\ m_T = 6 \cdot 10^{24}\ kg$

cherché: R_T

Solution: l'esquisse



Solution analytique

$$F_T = F_L \Rightarrow G\frac{mM_T}{R_T^2} = G\frac{mM_L}{R_L^2} \quad \Rightarrow \quad \sqrt{\frac{M_T}{M_L}} = \frac{R_T}{R_L}$$

$$\Rightarrow R_T = \left(r - R_T\right)\sqrt{\frac{M_T}{M_L}}$$

Solution numérique

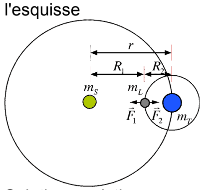
$$\Rightarrow \quad R_T = \frac{r\sqrt{\frac{M_T}{M_L}}}{1+\sqrt{\frac{M_T}{M_L}}} = \underline{\underline{3{,}5 \cdot 10^{8}\ m}}$$

2 donné:

$R_1 = 1{,}5 \cdot 10^{11}\ m\,;\ R_2 = 3{,}9 \cdot 10^{8}\ m\,;\ m_L = 7{,}3 \cdot 10^{22}\ kg\,;\ m_T = 6 \cdot 10^{24}\ kg\,;\ m_S = 2 \cdot 10^{30}\ kg$

cherché: $\frac{F_1}{F_2}$

Solution: l'esquisse



Solution analytique

$$\frac{F_1}{F_2} = \frac{G\frac{m_S m_L}{R_1^2}}{G\frac{m_T m_L}{R_2^2}} = \frac{m_S}{m_T}\frac{R_2^2}{R_1^2}$$

Solution numérique

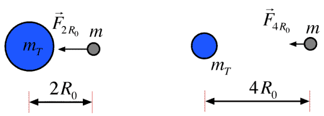
$$\underline{\underline{\frac{F_1}{F_2} = 2{,}25}}$$

3 donné:

$r = 2 \cdot R_0 \qquad F_{G_{2 \cdot R_0}} = 144000 N$

cherché: $F_{G_{4 \cdot R_0}}$

Solution: l'esquisse



Solution analytique

$$\frac{F_{G_{2R_0}}}{F_{G_{4R_0}}} = \frac{G\frac{m \cdot m_T}{(2R_0)^2}}{G\frac{m \cdot m_T}{(4R_0)^2}}$$

Solution numérique

$$\frac{F_{G_{2R_0}}}{F_{G_{4R_0}}} = \frac{16}{4} = 4 \Rightarrow \qquad \underline{\underline{F_{G_{4R_0}} = 36'000 N}}$$

4 donné: $P_{Terre} = 800\ N\,;\ g_{Mars} = 3{,}62\frac{m}{s^2}\,;\ g_{Terre} = 9{,}81\frac{m}{s^2}$

cherché: P_{Mars}

Solution: l'esquisse

Solution analytique

$$\left.\begin{array}{l} P_{Terre} = m \cdot g_{Terre} \\ P_{Mars} = m \cdot g_{Mars} \end{array}\right\} \Rightarrow m = \frac{P_{Terre}}{g_{Terre}} = \frac{P_{Mars}}{g_{Mars}}$$

$$\Rightarrow P_{Mars} = \frac{P_{Terre}}{g_{Terre}} \cdot g_{Mars}$$

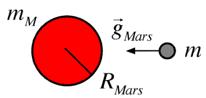
Solution numérique

$$\underline{\underline{P_{Mars} = 295{,}2N}}$$

5 donné: $m_{Mars} = 6.42 \cdot 10^{23}\ kg\,;\ g_{Mars} = 3{,}62\frac{m}{s^2}\,;\ G = 6{,}673231 \cdot 10^{-11}\frac{N \cdot m^2}{kg^2}$

cherché: R_{Mars}

Solution: l'esquisse



Solution analytique

$$F = G\frac{m_{Mars} m}{R_{Mars}^2} \wedge g_{Mars} = G\frac{m_{Mars}}{R_{Mars}^2}$$

$$\Rightarrow R_{Mars} = \sqrt{\frac{G \cdot m_{Mars}}{g_{Mars}}}$$

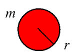
Solution numérique

$$\underline{\underline{R_{Mars} = 3440\ km}}$$

6 donné: $g = \frac{1}{2}g_0\,;\ r = \frac{1}{2}R_0\,;\ m_T = 6 \cdot 10^{24}\ kg$

cherché: m

Solution: l'esquisse



Solution analytique

$$\left.\begin{array}{l} g = \frac{Gm}{r^2} \\ g_0 = \frac{Gm_T}{R_0^2} \end{array}\right\} \wedge g = \frac{1}{2}g_0 \Rightarrow \frac{Gm}{r^2} = \frac{1}{2}\frac{Gm_T}{R_0^2}$$

$$\Rightarrow m = \frac{1}{2}\frac{m_T}{R_0^2}r^2 \wedge r = \frac{1}{2}R_0 \qquad \Rightarrow m = \frac{1}{2}\frac{m_T}{R_0^2}\left(\frac{1}{2}R_0\right)^2$$

$$\Rightarrow m = \frac{1}{8}m_T$$

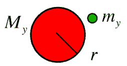
Solution numérique

$$\underline{\underline{m = 7{,}5 \cdot 10^{23}\ kg}}$$

7 donné: $g_0 = 9{,}81\,\frac{m}{s^2}\,;\, R_y = \frac{1}{2}R_0\,;\, M_y = \frac{1}{27}M_T\,;\, m = 70\ kg\,;\, M_T = 6\cdot 10^{24}\ kg$

cherché: P_y

Solution: l'esquisse



Solution analytique

$$P_y = g_y m = G\frac{M_y}{R_y^2}m = G\frac{1}{27}M_T m\frac{9}{R_T^2} = \frac{1}{3}g_0 m$$

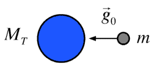
Solution numérique

$\underline{\underline{P_y = 228{,}9\ N}}$; $P_T = 686{,}7\ N$

8 donné: $g_0 = 9{,}81\,\frac{m}{s^2}\,;\, G = 6{,}673231\cdot 10^{-11}\,N\,\frac{m^2}{kg^2}\,;\, R_0 = 6{,}38\cdot 10^6\,m$

cherché: M_T

Solution: l'esquisse



Solution analytique

$$g_0 = G\frac{M_T}{R_0^2} \quad \Rightarrow M_T = \frac{g_0}{G}R_0^2$$

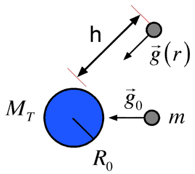
Solution numérique

$\underline{\underline{M_T = 5{,}98\cdot 10^{24}\,kg}}$

9 donné: $m = 50\ kg\,;\, h = 6380\ m\,;\, R_0 = 6380\cdot 10^3\,m\,;\, g_0 = 9{,}81\,\frac{m}{s^2}$

cherché: $a)\ P_0\,;\quad b)\ \frac{P_h}{P_0}$

Solution: l'esquisse



$$g(r) = G\frac{M_T}{r^2} = G\frac{M_T}{(R_0+h)^2} = g_0\frac{R_0^2}{(R_0+h)^2}$$

Solution analytique

$a)\ P_0 = m\cdot g_0$

$$b)\ \frac{P_h}{P_0} = \frac{mg_0\frac{R_0^2}{(R_0+h)^2}}{mg_0} = \frac{R_0^2}{(R_0+h)^2}$$

Solution numérique

$a)$ $\underline{\underline{P_0 = 490{,}5\ N}}$

$b)$ $\underline{\underline{\frac{P_h}{P_0} = 0{,}998}}$

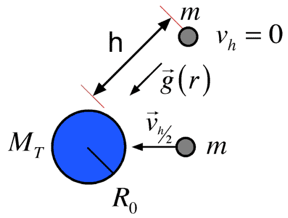
10 donné:

$g_0 = 9{,}81\,\frac{m}{s^2}\,;\, G = 6{,}673231\cdot 10^{-11}\,N\,\frac{m^2}{kg^2}\,;\, R_0 = 6{,}38\cdot 10^6\,m\,;\, M_T = 6\cdot 10^{24}\ kg$

$h = 10^4\ m$

cherché: $a)\ v_2 \quad ;\, b)\ h_{\frac{1}{2}v_2}$

Solution: l'esquisse



Solution analytique

Solution numérique

$a)\qquad \Delta E_{pot} + \Delta E_{cin} = 0$

$$\Rightarrow \left[-G\frac{M_T m}{R_0}\right] - \left[-G\frac{M_T m}{R_0+h}\right] + \frac{mv_2^2}{2} = 0$$

$$\Rightarrow v_2 = \sqrt{\frac{2}{m}\left[\frac{GM_T m}{R_0} - \frac{GM_T m}{R_0+h}\right]} = \sqrt{2GM_T\left[\frac{1}{R_0} - \frac{1}{R_0+h}\right]} \qquad \Rightarrow \underline{\underline{v_2 = 443\,\tfrac{m}{s}}}$$

$b)\qquad \Delta E_{pot} + \Delta E_{cin} = 0$

$$\Rightarrow \left[-G\frac{M_T m}{R_0+h_{\frac{1}{2}v}}\right] - \left[-G\frac{M_T m}{R_0+h}\right] + \frac{m\left(\frac{v_2}{2}\right)^2}{2} - \frac{mv_h^2}{2} = 0 \qquad \wedge \qquad v_h = 0$$

$$\Rightarrow \frac{1}{R_0+h_{\frac{1}{2}v}} - \frac{1}{R_0+h} = \frac{v_2^2}{8GM_T} \Rightarrow \frac{1}{R_0+h_{\frac{1}{2}v}} = \frac{1}{R_0+h} + \frac{v_2^2}{8GM_T}$$

$$\Rightarrow \frac{1}{R_0+h_{\frac{1}{2}v}} = \frac{8GM_T + v_2^2(R_0+h)}{(R_0+h)8GM_T}$$

$$\Rightarrow \frac{1}{R_0+h_{\frac{1}{2}v}} = \frac{8GM_T + 2GM_T\left[\frac{1}{R_0} - \frac{1}{R_0+h}\right](R_0+h)}{(R_0+h)8GM_T} = \frac{8 + 2\left(\frac{R_0+h-R_0}{R_0}\right)}{(R_0+h)8}$$

$$\Rightarrow h_{\frac{1}{2}v} = \frac{(R_0+h)}{R_0+\frac{1}{4}h}R_0 - R_0 = R_0\left(\frac{R_0+h}{R_0+\frac{1}{4}h} - 1\right) = R_0\left(\frac{4(R_0+h) - (4R_0+h)}{4R_0+h}\right)$$

$$\Rightarrow h_{\frac{1}{2}v} = \frac{3h}{4R_0+h}R_0 \qquad \Rightarrow \underline{\underline{h_{\frac{1}{2}v} = 2{,}5\ km}}$$

11 donné:

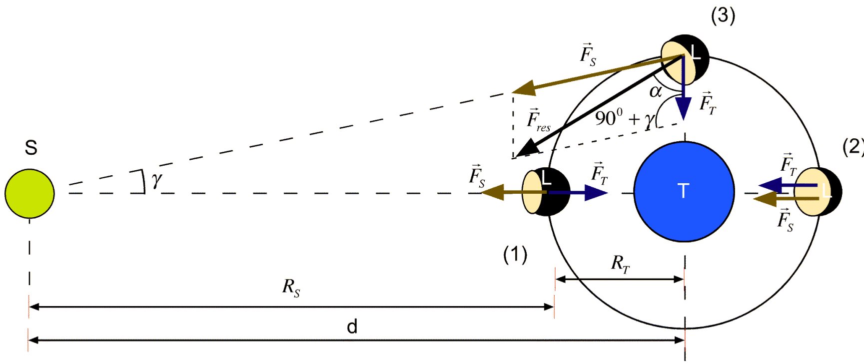
$d = 1{,}496 \cdot 10^{11} m$ $\quad M_S = 1{,}989 \cdot 10^{30}\ kg$

$R_T = 3{,}844 \cdot 10^{8} m$ $\quad M_T = 5{,}976 \cdot 10^{24}\ kg$

$M_M = 7{,}35 \cdot 10^{22}\ kg$

cherché: $a)\ F_{res_i}\ , i = 1,2,3$ $\quad b)\ raisons\ pour\ la\ situation\ (3)$

Solution: l'esquisse



Solution analytique Solution numérique

NOUVELLE LUNE (1)

$$F_{res} = F_S - F_T = GM_M\left(\frac{M_S}{R_S^2} - \frac{M_T}{R_T^2}\right) = \underline{\underline{2{,}398 \cdot 10^{20}\ N}}$$

PLAINE LUNE (2)

$$F_{res} = F_S + F_T = GM_M\left(\frac{M_S}{R_S^2} + \frac{M_T}{R_T^2}\right) = \underline{\underline{6{,}330 \cdot 10^{20}\ N}}$$

DEMIE LUNE (3)

$$F_{res} = \sqrt{F_T^2 + F_S^2 - 2F_T F_S \cos\left(90^0 + \gamma\right)}$$

$$\tan\gamma = \frac{R_T}{d} \qquad \Rightarrow \gamma = \arctan\frac{R_T}{d}$$

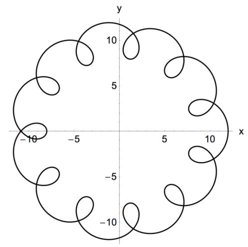
$$\frac{\sin\alpha}{\sin\left(90^0 + \gamma\right)} = \frac{F_S}{F_{res}} \qquad \Rightarrow \alpha = \arcsin\left[\sin\left(90^0 + \gamma\right) \cdot \tfrac{F_S}{F_{res}}\right]$$

$$\underline{\underline{F_{res} = 4{,}794 \cdot 10^{20}\ N}}\ ;\ \underline{\underline{\alpha = 65{,}4096^0}}$$

b)
Si l'on considère le mouvement de la lune et de la terre pendant une année, on constate que le mouvement est complexe c. à. d. un mouvement résultant d'une superposition de deux mouvement approximativement circulaires.

La trajectoire de la lune autour du soleil pendant une année

$$\alpha_T = \omega_T \cdot t$$
$$\alpha_L = \omega_L \cdot t$$
$$x(t) = R_T \cos\left(\omega_T \cdot t\right) + R_L \cos\left(\omega_L \cdot t\right)$$
$$y(t) = R_T \sin\left(\omega_T \cdot t\right) + R_L \sin\left(\omega_L \cdot t\right)$$



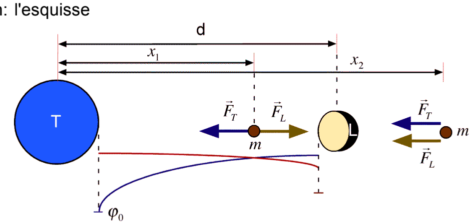
12 donné:

$d_{SE} = 1{,}496 \cdot 10^{11}\ m\ ;\ d_{EM} = 3{,}844 \cdot 10^{8}\ m\ ;\ R_E = 6{,}371 \cdot 10^{6}\ m$

$M_S = 1{,}989 \cdot 10^{30}\ kg\ ;\ M_E = 5{,}976 \cdot 10^{24}\ kg\ ;\ M_M = 7{,}35 \cdot 10^{22}\ kg\ ;\ G = 6{,}6732 \cdot 10^{-11}\ \frac{Nm^2}{kg^2}$

cherché: $F_M(x) = F_E(x) \Rightarrow a)\ \varphi_{res}(x) \quad b)\ \frac{\varphi_{res}}{\varphi_0}\ c)\ x$

Solution: l'esquisse



Solution analytique Solution numérique

$$a) \qquad F_T(x) = F_L(x) \Rightarrow G\frac{M_T m}{x^2} = G\frac{M_L m}{\left(d_{LT} - x\right)^2} \qquad \wedge\ d := d_{LT}$$

$$\Rightarrow M_L x^2 = M_T\left(d - x\right)^2 \qquad \Rightarrow x^2 = \frac{M_T}{M_L}\left(d - x\right)^2 \qquad \wedge\ c := \frac{M_T}{M_L}$$

$$\Rightarrow x^2 = c \cdot d^2 - 2dcx + cx^2 \qquad \Rightarrow x^2\left(1 - c\right) + 2c \cdot d \cdot x - cd^2 = 0$$

$$\Rightarrow x^2 + \tfrac{2c \cdot d}{(1-c)} \cdot x - \tfrac{cd^2}{(1-c)} = 0 \qquad \Rightarrow x_{1;2} = -\tfrac{c \cdot d}{(1-c)} \pm \sqrt{\tfrac{c^2 \cdot d^2 + cd^2(1-c)}{(1-c)^2}}$$

$$\underline{\underline{x_{1;2} = -\tfrac{c \cdot d}{(1-c)} \pm \sqrt{\tfrac{cd^2}{(1-c)^2}}}}$$

$$\Rightarrow \underline{\underline{x_1 = 3{,}46 \cdot 10^{8} m}} \qquad (entre\ Lune\ et\ la\ Terre)$$

$$\wedge \qquad \vec{F}_L = -\vec{F}_T \wedge F_L = F_T$$

$$\Rightarrow x_2 = 4{,}32 \cdot 10^{8} m \qquad (\grave{a}\ l'exterieur\ du\ segment\ joinant\ la\ Lune\ et\ la\ Terre)$$

$$\wedge \qquad \vec{F}_L = \vec{F}_T \wedge F_L = F_T$$

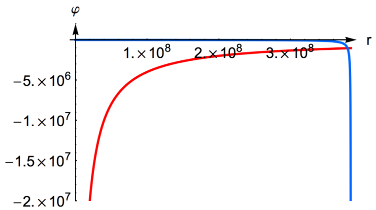
$$\varphi_{res} = \varphi_T + \varphi_L \qquad \varphi_T = -G\frac{M_T}{x_1} \wedge \qquad \varphi_L = -G\frac{M_L}{d - x_1}$$

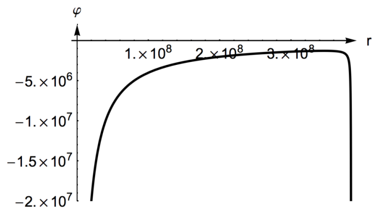
$$\Rightarrow \underline{\underline{\varphi_{res}(x_1) = -1{,}280 \cdot 10^{6}\ \tfrac{J}{kg}}} \qquad \Rightarrow \underline{\underline{\varphi_T(x_1) = -1{,}152 \cdot 10^{6}\ \tfrac{J}{kg}}} \qquad \Rightarrow \underline{\underline{\varphi_L(x_1) = -0{,}1278 \cdot 10^{6}\ \tfrac{J}{kg}}}$$

$$\Rightarrow \underline{\underline{\tfrac{\varphi_{res}}{\varphi_0} = 0{,}02}}$$

b) le maximum du potentiel résultant

le potentiel de la Terre et de la lune | le potentiel résultant



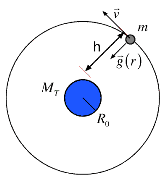


La position du point d'équilibre des Forces de gravitation se trouve approximativement $54\ R_0\quad (R_0 = rayon\ de\ la\ Terre)$ à part du centre de la terre et $6\ R_0$ du centre de la lune.

13 donné: $G\,;M_E\,;R_0\,;m\,;r$

cherché: E_{tot}^{sat}

Solution: l'esquisse



$$\varphi(r) = -G\frac{M_E}{r}$$

Solution analytique

$$E_{tot}^{sat} = GM_E m\left(\frac{1}{R_0} - \frac{1}{2r}\right)$$

Solution numérique

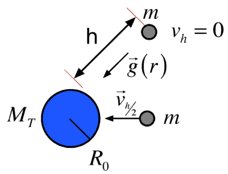
14 donné:

$$g_0 = 9{,}81\,\tfrac{m}{s^2}\,;G = 6{,}673231\cdot 10^{-11}\,N\,\tfrac{m^2}{kg^2}\,;R_0 = 6{,}38\cdot 10^{6}\,m\,;M_T = 6\cdot 10^{24}\ kg$$

$$h = 10000\ km = 10^7\ m$$

cherché: $a)\ v_2\quad ;b)\ h_{\frac{1}{2}v_2}$

Solution: l'esquisse



Solution analytique

Solution numérique

$a)\qquad \Delta E_{pot} + \Delta E_{cin} = 0$

$$\Rightarrow \left[-G\frac{M_T m}{R_0}\right] - \left[-G\frac{M_T m}{R_0+h}\right] + \frac{mv_2^2}{2} = 0$$

$$\Rightarrow v_2 = \sqrt{\frac{2}{m}\left[\frac{GM_T m}{R_0} - \frac{GM_T m}{R_0+h}\right]} = \sqrt{2GM_T\left[\frac{1}{R_0} - \frac{1}{R_0+h}\right]} \qquad \Rightarrow v_2 = 8734\ \tfrac{m}{s}$$

$b)\qquad \Delta E_{pot} + \Delta E_{cin} = 0$

$$\Rightarrow \left[-G\frac{M_T m}{R_0+h_{\frac{1}{2}v}}\right] - \left[-G\frac{M_T m}{R_0+h}\right] + \frac{m\left(\frac{v_2}{2}\right)^2}{2} - \frac{mv_h^2}{2} = 0 \qquad \wedge \qquad v_h = 0$$

$$\Rightarrow \frac{1}{R_0+h_{\frac{1}{2}v}} - \frac{1}{R_0+h} = \frac{v_2^2}{8GM_T} \Rightarrow \frac{1}{R_0+h_{\frac{1}{2}v}} = \frac{1}{R_0+h} + \frac{v_2^2}{8GM_T}$$

$$\Rightarrow \frac{1}{R_0+h_{\frac{1}{2}v}} = \frac{8GM_T + v_2^2(R_0+h)}{(R_0+h)8GM_T}$$

$$\Rightarrow \frac{1}{R_0+h_{\frac{1}{2}v}} = \frac{8GM_T + 2GM_T\left[\frac{1}{R_0} - \frac{1}{R_0+h}\right](R_0+h)}{(R_0+h)8GM_T} = \frac{8 + 2\left(\frac{R_0+h-R_0}{R_0}\right)}{(R_0+h)8}$$

$$\Rightarrow h_{\frac{1}{2}v} = \frac{(R_0+h)}{R_0+\frac{1}{4}h}R_0 - R_0 = R_0\left(\frac{R_0+h}{R_0+\frac{1}{4}h} - 1\right) = R_0\left(\frac{4(R_0+h)-(4R_0+h)}{4R_0+h}\right)$$

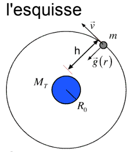
$$\Rightarrow h_{\frac{1}{2}v} = \frac{3h}{4R_0+h}R_0 \qquad \Rightarrow h_{\frac{1}{2}v} = 5386\ km$$

15 donné:

$g_0 = 9{,}81\,\frac{m}{s^2}\,;\, G = 6{,}673231\cdot10^{-11}\, N\,\frac{m^2}{kg^2}\,;\, R_0 = 6{,}38\cdot10^6\, m\,;\, M_T = 6\cdot10^{24}\, kg;\ h = 10000\, km = 10^7\, m$

cherché: $a)\, v;\quad b)\, T\,;\quad c)\,\frac{E}{m}$

Solution: l'esquisse



Solution analytique | Solution numérique

$$a)\, F_G = F_C \quad\Rightarrow\quad G\frac{M_T m}{(R_0+h)^2} = mr\omega^2 = m\frac{v^2}{r}$$

$$\Rightarrow v = \sqrt{\frac{GM_T}{R_0+h}} \qquad \underline{\underline{v = 7{,}35\cdot10^3\,\frac{m}{3}}}$$

$$b)\, v = \frac{2\pi(R_0+h)}{T} \qquad \Rightarrow T = \frac{2\pi(R_0+h)}{v} \qquad \underline{\underline{T = 6301\, s}}$$

$$c)\,\frac{E}{m} = \varphi(R_0+h) - \varphi_0 + \frac{v^2}{2} \qquad \underline{\underline{\frac{E}{m} = 3{,}55\cdot10^7\,\frac{J}{kg}}}$$

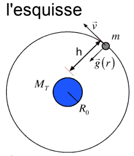
16 donné:

$g_0 = 9{,}81\,\frac{m}{s^2}\,;\, G = 6{,}673231\cdot10^{-11}\, N\,\frac{m^2}{kg^2}\,;\, R_0 = 6{,}38\cdot10^6\, m\,;\, M_T = 6\cdot10^{24}\, kg$

$a)\, h = 0\, m\,;\, b)\, h = R_0\,;\, c)\, h\to\infty$

cherché: $a)\, v_{h=0}\,;\left(\frac{E}{m}\right)_{h=0}\,;\, b)\, v_{h=R_0}\,;\left(\frac{E}{m}\right)_{h=R_0}\,;\ c)\, v_{fuite}\,;\left(\frac{E}{m}\right)_{fuite}$

Solution: l'esquisse



Solution analytique | Solution numérique

$$a)\, F_G = F_C \quad\Rightarrow\quad mg_0 = mr\omega^2 = m\frac{v^2}{R_0}$$

$$\Rightarrow v_{h=0} = \sqrt{g_0 R_0} \qquad \underline{\underline{v_{h=0} = 7906\,\frac{m}{3}}}$$

$$\left(\frac{E}{m}\right)_{h=0} = \frac{v^2}{2} \qquad \underline{\underline{\left(\frac{E}{m}\right)_{h=0} = 3{,}125\cdot10^7\,\frac{J}{kg}}}$$

$$b)\, F_G = F_C \quad\Rightarrow\quad G\frac{M_T m}{4R_0^2} = mr\omega^2 = m\frac{v^2}{2R_0}$$

$$v_{h=R_0} = \sqrt{\frac{GM_T}{2R_0}} \qquad \underline{\underline{v_{h=R_0} = 5594\,\frac{m}{3}}}$$

$$\left(\frac{E}{m}\right)_{h=R_0} = \varphi(2R_0) - \varphi_0 + \frac{v_{h=R_0}^2}{2} \qquad \underline{\underline{\left(\frac{E}{m}\right)_{h=R_0} = 4{,}69\cdot10^7\,\frac{J}{kg}}}$$

$$c)\, v_{fuite} = \sqrt{-2\varphi_0} \qquad \underline{\underline{v_{fuite} = 11'200\,\frac{m}{s}}}$$

$$\left(\frac{E}{m}\right)_{fuite} = \frac{v_{fuite}^2}{2} = \varphi_0 \qquad \underline{\underline{\left(\frac{E}{m}\right)_{fuite} = 6{,}27\cdot10^7\,\frac{J}{kg}}}$$

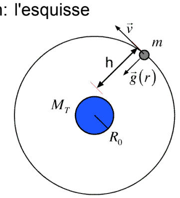
17 donné:

$g_0 = 9{,}81\,\frac{m}{s^2}\,;\, G = 6{,}673231\cdot10^{-11}\, N\,\frac{m^2}{kg^2}\,;\, R_0 = 6{,}38\cdot10^6\, m\,;\, M_T = 6\cdot10^{24}\, kg$

h

cherché: $a)\, E_{pot}(h) \qquad ;\, b)\, E_{cin}(h)$

Solution: l'esquisse



Solution analytique | Solution numérique

$$a)\, F_G = F_C \quad\Rightarrow\quad G\frac{M_T m}{(R_0+h)^2} = mr\omega^2 = m\frac{v^2}{R_0+h}$$

$$v_h = \sqrt{\frac{GM_T}{R_0+h}} \qquad \underline{\underline{E_{cin}(h) = \frac{m}{2}\cdot\frac{GM_T}{R_0+h}}}$$

$$b) \qquad \underline{\underline{E_{pot}(h) = -\frac{GM_T m}{R_0+h}}}$$

$$\underline{\underline{\frac{E_{cin}(h)}{E_{pot}(h)} = -\frac{1}{2}}}$$

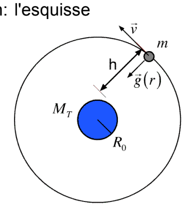
18 donné:

$g_0 = 9{,}81\,\frac{m}{s^2}\,;\, G = 6{,}673231\cdot10^{-11}\, N\,\frac{m^2}{kg^2}\,;\, R_0 = 6{,}38\cdot10^6\, m\,;\, M_T = 6\cdot10^{24}\, kg$

$m = 800\, kg\,;\, h_1 = 300\, km\,;\, h_2 = 250\, km$

cherché: $a)\, v_1\,;\, E_{cin_1}\,;\, E_{pot_1}\,;\, b)\, v_2\,;\, E_{cin_2}\,;\, E_{pot_2}\,;\, c)\,\Delta E_{pot}\,;\,\Delta E_{cin}$

Solution: l'esquisse



Solution analytique | Solution numérique

$$F_G = F_C \quad\Rightarrow\quad G\frac{M_T m}{(R_0+h)^2} = mr\omega^2 = m\frac{v^2}{R_0+h}$$

$$v_h = \sqrt{\frac{GM_T}{R_0+h}}$$

$$a) \qquad \underline{\underline{v_1 = 7728\,\frac{m}{s}}};\quad \underline{\underline{E_{cin_1} = 2{,}389\cdot10^{10}\, J}}\,;\, \underline{\underline{E_{pot_1} = 2{,}247\cdot10^9\, J}}$$

$$b) \qquad \underline{\underline{v_2 = 7757\,\frac{m}{s}}};\quad \underline{\underline{E_{cin_2} = 2{,}407\cdot10^{10}\, J}}\,;\, \underline{\underline{E_{pot_2} = 1{,}887\cdot10^9\, J}}$$

$$c) \qquad \underline{\underline{\Delta v = 29\,\frac{m}{s}}} \quad\Rightarrow \underline{\underline{\Delta E_{cin} = 1{,}8\cdot10^8\, J}}\,;\, \underline{\underline{\Delta E_{pot} = -3{,}6\cdot10^8\, J}}$$

$$\Rightarrow \underline{\underline{\Delta E_{pot} = -2\cdot\Delta E_{cin}}}$$