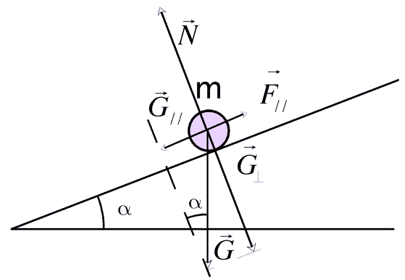
Solutions: le plan incliné et la quantité de mouvement

1 donné: $m = 10\,kg\,;\alpha = 30^0$

cherché: $F_{//}$

Solution: L'esquisse



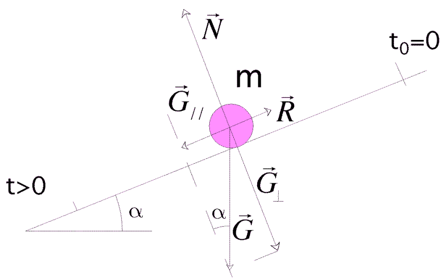
Solution analytique — solution numérique

$a = 0 \Rightarrow F_{//} = P_{//} \qquad \Rightarrow F_{//} = mg\sin\alpha \qquad \underline{\underline{F_{//} = 49{,}05\,N}}$

2 donné: $t = 10\,s\,;\alpha = 30^0\,;\mu = 0{,}5$

cherché: $a)\,v_E \;\; b)\,x$

Solution: L'esquisse



Solution analytique — solution numérique

$v(t) = at$

$x(t) = \frac{a}{2}t^2 \qquad \wedge v_0 = 0 \qquad \wedge x_0 = 0$

$F_{res} = ma = P_{//} - R = P\sin\alpha - \mu N = P\sin\alpha - \mu P\cos\alpha$

$\Rightarrow a = g(\sin\alpha - \mu\cos\alpha)$

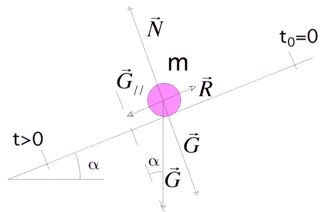
$\Rightarrow x(t) = \frac{g}{2}(\sin\alpha - \mu\cos\alpha)t^2 \qquad \underline{\underline{x(10s) = 32{,}86m}}$

$\Rightarrow v(t) = g(\sin\alpha - \mu\cos\alpha)t \qquad \underline{\underline{v(10s) = 6{,}57\,\tfrac{m}{s}}}$

3 donné: $x = 10\,m\,;\alpha = 30^0 \qquad \mu = 0$

cherché: $v(t)$

Solution: L'esquisse



Solution analytique — solution numérique

$v(t) = at$

$x(t) = \frac{a}{2}t^2 \qquad \wedge v_0 = 0 \qquad \wedge x_0 = 0 \qquad \Rightarrow t = \sqrt{\frac{2x(t)}{a}}$

$F_{res} = ma = P_{//} = P\sin\alpha$

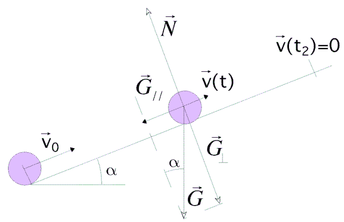
$\Rightarrow a = g(\sin\alpha)$

$\Rightarrow v(t) = g(\sin\alpha)t \qquad v(t) = \sqrt{g(\sin\alpha)2x(t)} \qquad \underline{\underline{v(t) = 9{,}9\,\tfrac{m}{s}}}$

4 donné: $m = 2\,kg\,;\alpha = 30^0\,;p_0 = mv_0 = 6\,\frac{kg\,m}{s} \quad v(t_2) = 0$

cherché: $x(t_2)$

Solution: L'esquisse



Solution analytique — solution numérique

$v(t_2) = -at_2 + v_0 = 0 \quad \Rightarrow t_2 = \frac{v_0}{a}$

$x(t_2) = \frac{-a}{2}t_2^2 + v_0 t_2 \qquad \wedge x_0 = 0 \qquad v_0 = \frac{p_0}{m} = 3\,\frac{m}{s}$

Déterminez a:

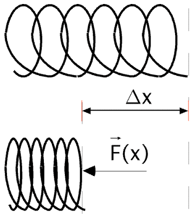
$F = ma = P_{//} = P\sin\alpha \qquad \Rightarrow a = g(\sin\alpha)$

$x(t_2) = \frac{-a}{2}t_2^2 + v_0 t_2 = \frac{-a}{2}\left(\frac{v_0}{a}\right)^2 + v_0\frac{v_0}{a} = \frac{v_0^2}{2a} = \frac{v_0^2}{2g(\sin\alpha)} \quad \Rightarrow \quad \underline{\underline{x(t_2) = 0{,}91m}}$

5 donné: $k = 50\,\frac{N}{m}\,;\Delta x = 10\,cm = 0{,}1\,m$

cherché: F

Solution: L'esquisse



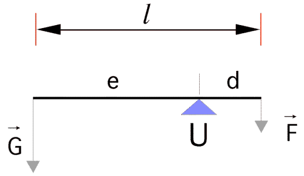
Solution analytique | solution numérique

$F = k \cdot \Delta x$ | $\underline{\underline{F = 5N}}$

6 donné: $l = 3\,m\,;\, m = 100\,kg\,;\, d = 1\,m$

cherché: F

Solution: L'esquisse



Solution analytique | solution numérique

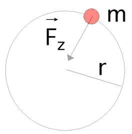
$\sum \vec{M} = \vec{0} \quad \Rightarrow P \cdot e = F \cdot d \quad \Rightarrow P \cdot (l - d) = F \cdot d$

$F = P\dfrac{l-d}{d}$ | $\underline{\underline{F = 1962N}}$

7 donné: $m = 0{,}2\,kg\,;\, f = 2s^{-1}\,;\, r = 0{,}5\,m$

cherché: F_C

Solution: L'esquisse



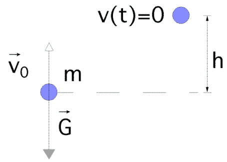
Solution analytique | solution numérique

$F_C = mr\omega^2 = mr(2\pi f)^2$ | $\underline{\underline{F_C = 15{,}8N}}$

8 donné: $m = 0{,}1\,kg\,;\, h = 20\,m\,;\, \Delta t = 0{,}5\,s\,;\, v(t) = 0$

cherché: $a)\ p_0 = mv_0 = F\Delta t \qquad b)\ F$

Solution: L'esquisse



Solution analytique | solution numérique

$p_0 = mv_0 = F\Delta t$

$h = x(t) = \dfrac{-g}{2}t^2 + v_0 t$

$v(t) = -gt + v_0 = 0 \qquad \Rightarrow t = \dfrac{v_0}{g}$

$h = \dfrac{-g}{2}\left(\dfrac{v_0}{g}\right)^2 + v_0\dfrac{v_0}{g} = \dfrac{v_0^2}{2g} \qquad \Rightarrow v_0 = \sqrt{2gh}$

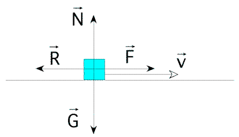
$a)\ p_0 = m\sqrt{2gh}$ | $\Rightarrow \underline{\underline{p_0 = 1{,}98\ \tfrac{kg \cdot m}{s}}}$

$b)\ F = -\dfrac{mv_0}{\Delta t}$ | $\Rightarrow \underline{\underline{F = -3{,}96\ N}}$

9 donné: $m = 50\ kg\ ;\ \mu = 0,1\ ;\ a = 0 \Rightarrow v = konst$

cherché: F

Solution: L'esquisse



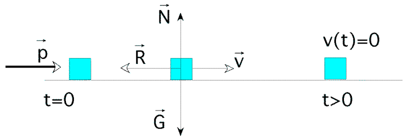
Solution analytique — solution numérique

$F = R = \mu N = \mu mg$ — $\underline{\underline{F = 49,5N}}$

10 donné: $m = 10\ kg\ ;\ \Delta t = \frac{1}{10}s\ ;\ F = 10N\ ;\ \mu = 0,1\ ;\ \alpha = 0\ ;\ v(t) = 0$

cherché: $x(t)$

Solution: L'esquisse



Solution analytique — solution numérique

$p = F\Delta t = mv_0 \quad \Rightarrow v_0 = \frac{F\Delta t}{m}$

$Déterminez\ a:$

$R = ma = \mu mg \quad \Rightarrow a = \mu g \quad (nach\ links)$

$x(t) = \frac{-a}{2}t^2 + v_0 t \quad \Rightarrow t = \frac{v_0}{a}$

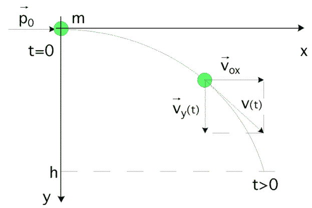
$v(t) = -at + v_0 = 0$

$x(t) = \frac{-\mu g}{2}\left(\frac{v_0}{\mu g}\right)^2 + v_0\frac{v_0}{\mu g} = \frac{v_0^2}{2\mu g} \qquad \Rightarrow \underline{\underline{x(t) = 0,0051m \approx 5mm}}$

11 donné: $m = 0,2\ kg\ ;\ h = 45\ m\ ;\ l = 30\ m$

cherché: $p_0 = mv_{0X} = F\Delta t$

Solution: L'esquisse



Solution analytique — solution numérique

$direction\ x: \quad x(t) = l = v_{0X}t \quad \wedge \quad v_{0X} = konst$

$direction\ y: \quad y(t) = \frac{g}{2}t^2 + v_{0Y}\cdot t = h \wedge \quad v_{0Y} = 0$

$v_y(t) = g\cdot t$

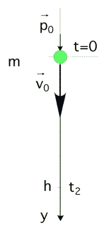
$t = \frac{l}{v_{0X}} \Rightarrow \quad y(t) = \frac{g}{2}\left(\frac{l}{v_{0X}}\right)^2 = h \quad \Rightarrow v_{0X} = \sqrt{\frac{g}{2h}}\cdot l$

$\Rightarrow p_0 = m\cdot v_{0X} = m\sqrt{\frac{g}{2h}}\cdot l \qquad \Rightarrow \underline{\underline{p_0 = 1,98\frac{kg\cdot m}{s}}}$

12 donné: $m = 0,2\ kg\ ;\ h = 45m\ ;\ t_2 = 1s$

cherché: p_0

Solution: L'esquisse



Solution analytique — solution numérique

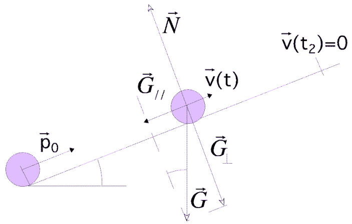
$y(t_2) = h = \frac{g}{2}t^2 + v_0\cdot t$

$v_0 = \frac{h}{t_2} - \frac{g}{2}t_2 \quad \Rightarrow p_0 = m\left(\frac{h}{t_2} - \frac{g}{2}t_2\right) \qquad \Rightarrow \underline{\underline{p_0 = 8,02\frac{kg\cdot m}{s}}}$

13 donné: $m = 10\ kg\ ;\alpha = 30^0\ ;l = 10\ m$

cherché: p_0

Solution: L'esquisse



Solution analytique — solution numérique

$$x(t) = l = \frac{-a}{2}t^2 + v_0 t$$

$$a = g\sin\alpha$$

$$x(t) = l = \frac{-g\sin\alpha}{2}t^2 + v_0 t$$

$$v(t) = g\cdot t\cdot\sin\alpha + v_0 = 0 \qquad \Rightarrow t = \frac{v_0}{g\sin\alpha}$$

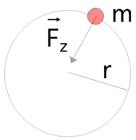
$$\Rightarrow l = \frac{-g\sin\alpha}{2}\left(\frac{v_0}{g\sin\alpha}\right)^2 + v_0\frac{v_0}{g\sin\alpha} = \frac{v_0^2}{2g\sin\alpha}$$

$$\Rightarrow v_0 = \sqrt{2\lg(\sin\alpha)} \qquad \Rightarrow p_0 = m\sqrt{2\lg(\sin\alpha)} \qquad \Rightarrow \underline{\underline{p_0 = 99{,}05\,\tfrac{kg\cdot m}{s}}}$$

14 donné: $m = 1\ kg\ ; l = 2\ m\ ; T = 0{,}5\ s$

cherché: F_C

Solution: L'esquisse



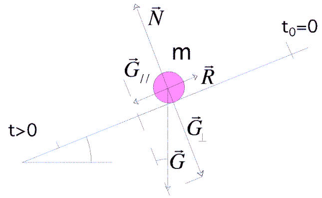
Solution analytique — solution numérique

$$F_C = ml\omega^2 = ml\left(2\pi\frac{1}{T}\right) \qquad \underline{\underline{F_C = 315{,}83\,N}}$$

15 donné: $\mu = 0{,}5\ ;\alpha = 45^0\ ;t = 4\ s$

cherché: $a)\ v \qquad b)\ x(t)$

Solution: L'esquisse



Solution analytique — solution numérique

$$v(t) = at$$

$$x(t) = \frac{a}{2}t^2 \qquad \wedge v_0 = 0 \qquad \wedge x_0 = 0$$

$$F_{res} = ma = P_{//} - R = P\sin\alpha - \mu N = P\sin\alpha - \mu P\cos\alpha$$

$$\Rightarrow a = g(\sin\alpha - \mu\cos\alpha)$$

$$\Rightarrow x(t) = \frac{g}{2}(\sin\alpha - \mu\cos\alpha)t^2 \qquad \underline{\underline{x(4s) = 27{,}75m}}$$

$$\Rightarrow v(t) = g(\sin\alpha - \mu\cos\alpha)t \qquad \underline{\underline{v(4s) = 13{,}97\,\tfrac{m}{s}}}$$

16 donné : m, v, réflexion
cherché : F
Solution

$$F = \frac{\Delta p}{\Delta t} = \frac{2mv}{t}$$

17 donné : $m = 0.15kg; \quad v_1 = 40\ \tfrac{m}{s}; \quad v_2 = -60\ \tfrac{m}{s}; \quad \Delta t = 5\ ms$
cherché : F
Solution :

$$F = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{0{,}15kg100\tfrac{m}{s}}{5\cdot 10^{-3}s} = 3000N$$

18 donné : $F = 50N; \quad \Delta t = 10\ ms; \quad m_B = 0{,}2\ kg$
cherché : Δv
Solution: $\Delta p = m\Delta v = F\Delta t \quad \Rightarrow \Delta v = \frac{F\Delta t}{m} = \frac{50N\ 0{,}01s}{0{,}2kg} \quad \Rightarrow \quad \Delta v = 2{,}5\tfrac{m}{s}$

19 donné : $m = 10\ kg; \quad F(t) = \frac{t}{4}50; \quad v_1 = 0$
cherché : v_2
Solution :

$$v_2 = v(t) = \int_0^{4s} a(t)dt \quad \wedge \quad a(t) = \frac{F(t)}{m} = t\frac{50N}{4s\cdot 10\ kg}$$

$$\Rightarrow \quad v_2 = v(t) = \frac{5N}{4s\cdot kg}\int_0^{4s} t\cdot dt = \frac{5N}{4s\cdot kg}\left[\frac{t^2}{2}\right]_0^{4s} = \frac{5N}{4s\cdot kg}\left[\frac{16s^2}{2}\right] \quad \Rightarrow \quad v_2 = 10\tfrac{m}{s}$$

20 donné : $m_1 = 0{,}01\ kg; \quad m_2 = 2\ kg; \quad h = 0{,}12\ m; \quad$ *collistion inélastique*
cherché : v
Solution :

$$P: \quad m_1 v = (m_1 + m_2)u \quad \Rightarrow \quad u = \frac{m_1 v}{(m_1 + m_2)} \quad [1]$$

$$E: \quad m_1\frac{v^2}{2} = (m_1 + m_2)\frac{u^2}{2} + \Delta Q = (m_1 + m_2)g\cdot h + \Delta Q$$

$$\Rightarrow \quad \frac{u^2}{2} = g\cdot h \quad avec \quad [1] \quad \frac{m_1^2 v^2}{(m_1 + m_2)^2} = 2g\ h$$

$$\Rightarrow \quad v$$

$$\Rightarrow \quad v = \sqrt{\frac{2(m_1 + m_2)^2 g\cdot h}{m_1^2}} \quad v = \sqrt{\frac{2(2{,}01kg)^2 9{,}81\tfrac{m}{s^2}0{,}12m}{(0{,}01kg)^2}} \quad \boxed{v = 308{,}4\tfrac{m}{s}}$$

21 donné : $m_1 = 2kg; \quad v_2 = 0; \quad u_1 = \tfrac{1}{4}v_1$
cherché : m_2
Solution :

$$p: \quad m_1 v_1 = m_1 u_1 + m_2 u_2 \quad \Rightarrow \quad 2v_1 = \tfrac{1}{2}v_1 + m_2 u_2 \quad \Rightarrow \quad \tfrac{3}{2}v_1 = m_2 u_2 \quad [1]$$

$$E: \quad m_1 v_1^2 = m_1 u_1^2 + m_2 u_2^2 \quad \Rightarrow \quad 2v_1^2 = \tfrac{1}{8}v_1^2 + m_2 u_2^2 \quad \Rightarrow \quad \tfrac{15}{8}v_1^2 = m_2 u_2^2 \quad [2]$$

$$[1]\rightarrow \quad \tfrac{9}{4}v_1^2 = m_2^2 u_2^2 \quad \wedge \quad \tfrac{15}{8}v_1^2 = m_2 u_2^2 \quad \Rightarrow \tfrac{9}{4}\cdot\tfrac{8}{15}kg = m_2 \quad \boxed{m_2 = \tfrac{6}{5}kg}$$

22 donné : $m_1 = 4{,}5\cdot 10^{-3}kg; \quad m_2 = 1{,}8\ kg; \quad v_2 = 0; \ \mu = 0.2; \quad d = 1{,}8\ m$
cherché : v_1
Solution :

$$P: \quad m_1 v_1 = (m_1 + m_2)u_1 \quad [1]$$

$$d = x(t) = -\frac{a}{2}t^2 + u_1 t \quad [2]$$

$$v(t) = u_2 = 0 = -at + u_1 \quad \Rightarrow t = \tfrac{u_1}{a} \quad [3]$$

$$ma = R_t = \mu N = \mu mg \quad \Rightarrow a = \mu g \quad [4]$$

$$[3]\rightarrow[2] \quad \Rightarrow d = -\frac{a}{2}\left(\tfrac{u_1}{a}\right)^2 + u_1\tfrac{u_1}{a} = \frac{u_1^2}{2a} \overset{[4]}{=} \frac{u_1^2}{2\mu g} \quad \Rightarrow u_1 = \sqrt{2d\mu g}$$

$$[1]\Rightarrow \quad v_1 = \frac{m_1 + m_2}{m_1}u_1 \quad \Rightarrow \quad \boxed{v_1 = 1065{,}7\tfrac{m}{s}}$$

23 donné : $m_1; m_2; h$
cherché : h_2
Solution :

$$E_{pot} = m_1 gh = m\frac{v^2}{2} = (m_1 + m_2)gh_2 \quad \Rightarrow \quad \boxed{h_2 = \frac{m_1}{(m_1 + m_2)}h}$$

24 donné : $m_1; m_2; h$
cherché : h_2
Solution :

$$(2m_1 a + m_1 a)\frac{\Delta l^2}{2} = 60J \quad \Rightarrow \quad 3m_1 a\frac{\Delta l^2}{2} = 60J \quad \frac{\Delta W_1}{\Delta W_2} = \frac{1}{2}$$

$$\Rightarrow \quad \boxed{\Delta W_1 = 40J \quad \Delta W_2 = 20J}$$

25 donné : $m_2 = 1840m_1$
cherché : $\frac{E_H}{E_e}$
Solution :

$$P: \quad v_1 = -u_1 + 1840u_2 \quad E: \quad v_1^2 = u_1^2 + 1840u_2^2$$

$$\Rightarrow \quad \frac{E_H}{E_e} = \frac{\tfrac{1}{2}1840\ u_2^2}{\tfrac{1}{2}v_1^2} \quad \Rightarrow \quad \boxed{\frac{E_H}{E_e} = 0{,}0022}$$

26 donné : $m_1 = 2kg; \ v_1 = 10\tfrac{m}{s}; \quad m_2 = 10kg; \quad v_2 = 3\tfrac{m}{s}; \quad k = 1120N$
cherché : $x = \Delta x_{max}$
Solution :

$$P: \quad m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2 = (m_1 + m_2)u \quad \Rightarrow \quad u = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \quad [1]$$

$$E: \quad \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} = \frac{m_1 u_1^2}{2} + \frac{m_2 u_2^2}{2} = k\frac{x^2}{2} + \frac{(m_1 + m_2)}{2}\cdot u^2 \quad [2]$$

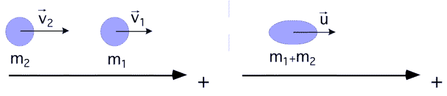
$$[1]\Rightarrow \quad u = 5\tfrac{m}{s}$$

$$[2]\Rightarrow \quad x = \sqrt{\frac{1}{k}\left(m_1 v_1^2 + m_2 v_2^2 - (m_1 + m_2)u^2\right)} \quad \Rightarrow \quad \boxed{x = 0{,}25\ m}$$

16 donné: $m = 0{,}4\ kg\,;\ v_1 = 8\ \frac{m}{s}\,;\ m_2 = 0{,}6\ kg\,;\ v_2 = 12\ \frac{m}{s}$ *inel. Stoss*

cherché: u

Solution: L'esquisse



Solution analytique — solution numérique

conservation de la quantité de mouvement:

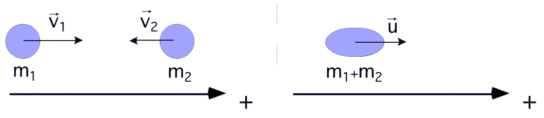
$$m_1v_1 + m_2v_2 = (m_1+m_2)\cdot u$$

$$\Rightarrow u = \frac{m_1v_1+m_2v_2}{(m_1+m_2)} \qquad \underline{\underline{u = 10{,}4\ \tfrac{m}{s}}}$$

17 donné: $m_1\,;\ m_2\,;\ v_1\,;\ v_2$

cherché: $\Delta Q^{\uparrow}$

Solution: L'esquisse



Solution analytique — solution numérique

conservation de l'énergie et de la quantité de mouvement:

$$E: \quad \frac{m_1}{2}v_1^2 + \frac{m_2}{2}v_2^2 = \frac{(m_1+m_2)}{2}u^2 + \Delta Q^{\uparrow}$$

$$p: \quad m_1v_1 - m_2v_2 = (m_1+m_2)\cdot u$$

$$\Rightarrow u = \frac{m_1v_1 - m_2v_2}{(m_1+m_2)} \quad \Rightarrow \quad u^2 = \left(\frac{m_1v_1-m_2v_2}{(m_1+m_2)}\right)^2$$

$$\Rightarrow \Delta Q^{\uparrow} = \frac{m_1}{2}v_1^2 + \frac{m_2}{2}v_2^2 - \frac{(m_1+m_2)}{2}u^2$$

$$\Rightarrow \Delta Q^{\uparrow} = \frac{1}{2}\left[m_1v_1^2 + m_2v_2^2 - (m_1+m_2)\left(\frac{m_1v_1-m_2v_2}{(m_1+m_2)}\right)^2\right]$$

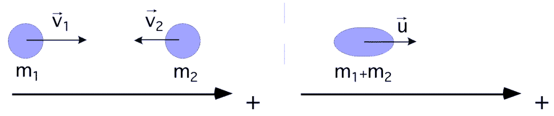
$$\Rightarrow \Delta Q^{\uparrow} = \frac{m_1^2v_1^2 + m_1m_2v_2^2 + m_2^2v_2^2 - m_1^2v_1^2 + 2m_1m_2v_1v_2 - m_2^2v_2^2}{2(m_1+m_2)}$$

$$\Rightarrow \Delta Q^{\uparrow} = \frac{m_1m_2(v_1^2+v_2^2) + 2m_1m_2v_1v_2}{2(m_1+m_2)} \qquad \Rightarrow \underline{\underline{\Delta Q^{\uparrow} = \frac{m_1m_2(v_1+v_1)^2}{2(m_1+m_2)}}}$$

18 donné: $(m_1+m_2) = 20kg\,;\ \vec v_1 = -k\vec v_2\,;\ v_1 = 4\,\frac{m}{s}\,;\ v_2 = 12\,\frac{m}{s}\,;$ *unel Stoss*$\,;\ u = 3\,\frac{m}{s}\,;\ \vec u = l\cdot\vec v_1$

cherché: $m_1\,;\ m_2\,;\ \Delta Q^{\uparrow}$

Solution: L'esquisse



Solution analytique — solution numérique

conservation de l'énergie et de la quantité de mouvement:

$$[1]\,E: \quad \frac{m_1}{2}v_1^2 + \frac{m_2}{2}v_2^2 = \frac{(m_1+m_2)}{2}u^2 + \Delta Q^{\uparrow} \quad \overset{N17}{\Rightarrow} \quad \underline{\underline{\Delta Q^{\uparrow} = \frac{m_1m_2(v_1+v_1)^2}{2(m_1+m_2)}}}$$

$$[2]\,p: \quad m_1v_1 - m_2v_2 = (m_1+m_2)\cdot u$$

$$[3] \quad m_1 + m_2 = m \quad \Rightarrow m_2 = m - m_1$$

$$[3]\to[1] \Rightarrow m_1v_1 - (m-m_1)v_2 = (m_1+m_2)\cdot u \qquad \Rightarrow m_1(v_1+v_2) - mv_2 = (m_1+m_2)\cdot u$$

$$\underline{\underline{\Rightarrow m_1 = \frac{(m_1+m_2)\cdot u + mv_2}{v_1+v_2}}} \qquad \Rightarrow \qquad \underline{\underline{m_1 = 18{,}75\ kg}}$$

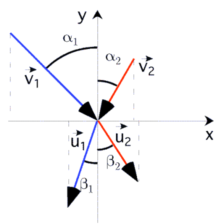
$$\underline{\underline{\Rightarrow m_2 = m - m_1}} \qquad \Rightarrow \qquad \underline{\underline{m_2 = 1{,}25\ kg}}$$

$$\underline{\underline{\Rightarrow \Delta Q^{\uparrow} = \frac{m_1m_2(v_1+v_1)^2}{2(m_1+m_2)}}} \qquad \Rightarrow \qquad \underline{\underline{\Delta Q^{\uparrow} = 150\ J}}$$

19 donné: $m_1 = 20\ kg\,;\ m_2 = 10\ kg\,;\ v_1 = 2\,\frac{m}{s}\,;\ v_2 = 1\,\frac{m}{s}\,;\ \alpha_1 = 45^0\,;\ \alpha_2 = 30^0$

cherché: $\beta_1\,;\ \beta_2\,;\ u_1\,;\ u_2$

Solution: L'esquisse



L'axe des x est l'axe central de la collision. L'énergie et la quantité de mouvement est échangé que en direction de l'axe des x.

$u_{y1} = v_{y1}; \qquad u_{y2} = v_{y2}$

De plus on a la conservation de l'énergie et de la quantité de mouvement en direction de l'axe des x:

$$E_x: \quad m_1v_{x1}^2 + m_2v_{x2}^2 = m_1u_{1x}^2 + m_2u_{2x}^2 \qquad 1)$$

$$p_x: \quad m_1v_{x1} - m_2v_{x2} = -m_1u_{1x} + m_2u_{2x} \qquad 2)$$

$E_x:\quad 2v_{x1}^2+v_{x2}^2=2u_{x1}^2+u_{x2}^2 \qquad 1)$

$p_x:\quad 2v_{x1}-v_{x2}=-2u_{x1}+u_{x2} \qquad 2) \qquad \Rightarrow u_{x2}=2u_{x1}+2v_{x1}-v_{x2} \qquad \to 1)$

$\Rightarrow\quad 2v_{x1}^2+v_{x2}^2=2u_{x1}^2+\left(2u_{x1}+2v_{x1}-v_{x2}\right)^2 \qquad 3)$

Calcul intermédiaire: Le trinôme au carré:

$$(2a+2b-c)(2a+2b-c)= \begin{array}{cccccc} 4a^2 & +4ab & -2ac & & & \\ & +4ab & & +4b^2 & -2bc & \\ & & -2ac & & -2bc & +c^2 \\ \hline \end{array}$$

$$(2a+2b-c)(2a+2b-c)= 4a^2 \quad +8ab \quad -4ac \quad +4b^2-4bc \quad +c^2$$

De l'équation 3) on a :

$\Rightarrow\quad 2v_{x1}^2+v_{x2}^2=2u_{x1}^2+4u_{x1}^2+8u_{x1}v_{x1}-4u_{x1}v_{x2}+4v_{x1}^2-4v_{x1}v_{x2}+v_{x2}^2$

$\Rightarrow\quad 6u_{x1}^2+\left(8v_{x1}-4v_{x2}\right)u_{x1}-4v_{x1}v_{x2}+2v_{x1}^2=0$

$\Rightarrow\quad u_{x1}^2+\left(\frac{4}{3}v_{x1}-\frac{2}{3}v_{x2}\right)u_{x1}-\frac{3}{2}v_{x1}v_{x2}+\frac{1}{3}v_{x1}^2=0$

Nous déterminons la solution de l'équation quadratique:

$\Rightarrow\quad u_{x1}=-\left(\frac{2}{3}v_{x1}-\frac{1}{3}v_{x2}\right)\pm\sqrt{\left(\frac{2}{3}v_{x1}-\frac{1}{3}v_{x2}\right)^2+\frac{3}{2}v_{x1}v_{x2}-\frac{1}{3}v_{x1}^2}$

$\Rightarrow\quad u_{x1}=-\left(\frac{2}{3}v_{x1}-\frac{1}{3}v_{x2}\right)\pm\sqrt{\frac{4}{9}v_{x1}^2-\frac{4}{9}v_{x1}v_{x2}+\frac{1}{9}v_{x2}^2+\frac{3}{2}v_{x1}v_{x2}-\frac{1}{3}v_{x1}^2}$

$\Rightarrow\quad u_{x1}=-\left(\frac{2}{3}v_{x1}-\frac{1}{3}v_{x2}\right)\pm\sqrt{\frac{1}{9}v_{x1}^2+\frac{2}{9}v_{x1}v_{x2}+\frac{1}{9}v_{x2}^2}$

$\Rightarrow\quad u_{x1}=-\left(\frac{2}{3}v_{x1}-\frac{1}{3}v_{x2}\right)\pm\frac{1}{3}\sqrt{v_{x1}^2+2v_{x1}v_{x2}+v_{x2}^2}$

$\Rightarrow\quad u_{x1}=-\left(\frac{2}{3}v_{x1}-\frac{1}{3}v_{x2}\right)\pm\frac{1}{3}\left(v_{x1}+v_{x2}\right)$

D'ou la première solution s'écrit:

$\Rightarrow\quad u_{x1}=-\left(\frac{2}{3}v_{x1}-\frac{1}{3}v_{x2}\right)+\frac{1}{3}\left(v_{x1}+v_{x2}\right)$

$\Rightarrow\quad u_{x1}=-\frac{1}{3}v_{x1}+\frac{2}{3}v_{x2} \qquad \wedge \qquad u_{x2}=-\frac{2}{3}v_{x1}+\frac{4}{3}v_{x2}+2v_{x1}-v_{x2}$

$\Rightarrow\quad \underline{\underline{u_{x1}=\frac{1}{3}\left(2v_{x2}-v_{x1}\right)}} \qquad \wedge \qquad \underline{\underline{u_{x2}=\frac{1}{3}\left(4v_{x1}+v_{x2}\right)}}$

ou la deuxième solution:

$\Rightarrow\quad u_{x1}=-\left(\frac{2}{3}v_{x1}-\frac{1}{3}v_{x2}\right)-\frac{1}{3}\left(v_{x1}+v_{x2}\right)$

$\Rightarrow\quad u_{x1}=-v_{x1} \qquad \wedge \qquad u_{x2}=-2v_{x1}+2v_{x1}-v_{x2}$

$\Rightarrow\quad \underline{\underline{u_{x1}=-v_{x1}}} \qquad \wedge \qquad \underline{\underline{u_{x2}=-v_{x2}}}$

Solution numérique

$\sin\alpha_1=\sin 45^0=\frac{1}{\sqrt{2}} \quad ; \quad \sin\alpha_2=\sin 30^0=\frac{1}{2}$

$\Rightarrow v_{x1}=v_1\sin 45^0=\frac{2}{\sqrt{2}}\frac{m}{s}=\sqrt{2}\frac{m}{s} \quad ; \quad v_{x2}=v_2\sin 30^0=\frac{1}{2}\frac{m}{s}$

$\Rightarrow v_{y1}=v_1\cos 45^0=\frac{2}{\sqrt{2}}\frac{m}{s}=\sqrt{2}\frac{m}{s} \quad ; \quad v_{y2}=v_2\cos 30^0=\sqrt{\frac{3}{4}}\frac{m}{s}$

d'ou la première solution:

$u_{x1}=\frac{1}{3}\left(1-\sqrt{2}\right)\frac{m}{s} \quad ; \quad u_{x2}=\frac{1}{6}\left(8\sqrt{2}+1\right)\frac{m}{s}$

$u_{y1}=v_{y1}=v_1\cos\alpha_1=2\sqrt{2}\frac{m}{s} \quad ; \quad u_{y2}=v_1\sin\alpha_2=\sqrt{\frac{3}{4}}\frac{m}{s}$

$\beta_1=\arctan\frac{\sqrt{2}-1}{3\sqrt{2}}=5{,}5762^0 \quad ; \quad \beta_2=\arctan\frac{1+8\sqrt{2}}{3\sqrt{3}}=67{,}1212^0$

et la deuxième solution

$u_{x1}=-\sqrt{2}\frac{m}{s} \quad ; \quad u_{x2}=-\frac{1}{2}\frac{m}{s}$

$u_{y1}=v_{y1}=v_1\cos\alpha_1=2\sqrt{2}\frac{m}{s} \quad ; \quad u_{y2}=v_1\sin\alpha_2=\sqrt{\frac{3}{4}}\frac{m}{s}$

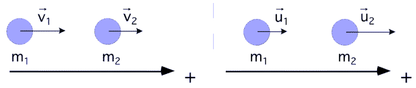
$\beta_1=\arctan 1=45^0 \quad ; \quad \beta_2=\arctan\frac{\frac{1}{2}}{\sqrt{\frac{3}{4}}}=30^0$

Interprétation:

La première solution donne la situation après la collision. La deuxième solution est la situation avant la collision.

20 donné: $P:\ m_1v_1+m_2v_2=m_1u_1+m_2u_2 \qquad E:\frac{m_1v_1^2}{2}+\frac{m_2v_2^2}{2}=\frac{m_1u_1^2}{2}+\frac{m_2u_2^2}{2}$

cherché: Démonstration: $\left|u_2-u_1\right|=\left|v_2-v_1\right| \qquad \forall m_1,m_2$



Solution: Démonstration

$E:\frac{m_1v_1^2}{2}+\frac{m_2v_2^2}{2}=\frac{m_1u_1^2}{2}+\frac{m_2u_2^2}{2} \qquad \Rightarrow m_1v_1^2+m_2v_2^2=m_1u_1^2+m_2u_2^2$

$\Rightarrow m_1\left(v_1^2-u_1^2\right)=m_2\left(u_2^2-v_2^2\right)$

$P:\ m_1v_1+m_2v_2=m_1u_1+m_2u_2 \Rightarrow m_1\left(v_1-u_1\right)=m_2\left(u_2-v_2\right)$

$\Rightarrow\frac{m_1}{m_2}=\frac{\left(u_2^2-v_2^2\right)}{\left(v_1^2-u_1^2\right)}=\frac{m_1}{m_2}=\frac{\left(u_2-v_2\right)}{\left(v_1-u_1\right)} \quad \Rightarrow\frac{\left(u_2-v_2\right)\left(u_2+v_2\right)}{\left(v_1-u_1\right)\left(v_1+u_1\right)}=\frac{\left(u_2-v_2\right)}{\left(v_1-u_1\right)}$

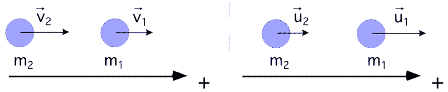
$\Rightarrow\frac{\left(u_2+v_2\right)}{\left(v_1+u_1\right)}=1 \qquad \Rightarrow u_2+v_2=v_1+u_1 \qquad \Rightarrow u_2-u_1=v_1-v_2=-\left(v_2-v_1\right)$

$\Rightarrow\underline{\underline{\left|u_2-u_1\right|=\left|v_2-v_1\right|}}$

21 donné: $m_1 = 0{,}6\,kg\,; m_2 = 0{,}4\,kg\,; u_1 = 4\,\frac{m}{s}\,; u_2 = 1\,\frac{m}{s}\,; el\,Stoss$

cherché: $v_1\,; v_2$

Solution: L'esquisse



Conservation de l'énergie et de la quantité de mouvement:

$$E: \quad \frac{m_1}{2}v_1^2 + \frac{m_2}{2}v_2^2 = \frac{m_1}{2}u_1^2 + \frac{m_2}{2}u_2^2$$

$$p: \quad m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2 \qquad \Rightarrow v_1 = \frac{m_1u_1 + m_2(u_2 - v_2)}{m_1} = \frac{m_1u_1 + m_2(u_2 - v_2)}{m_1}$$

$$\Rightarrow \frac{m_1}{2}\left[\frac{u_1m_1 + m_2(u_2 - v_2)}{m_1}\right]^2 + \frac{m_2}{2}v_2^2 = \frac{m_1}{2}u_1^2 + \frac{m_2}{2}u_2^2$$

$$\Rightarrow \frac{\left[u_1m_1 + m_2(u_2 - v_2)\right]^2}{m_1} + m_2v_2^2 = m_1u_1^2 + m_2u_2^2$$

$$\Rightarrow m_1u_1^2 + 2m_2u_1(u_2 - v_2) + \frac{m_2^2}{m_1}(u_2 - v_2)^2 + m_2v_2^2 = m_1u_1^2 + m_2u_2^2$$

$$\Rightarrow 2m_2u_1u_2 - 2m_2u_1v_2 + \frac{m_2^2}{m_1}u_2^2 - 2\frac{m_2^2}{m_1}u_2v_2 + \frac{m_2^2}{m_1}v_2^2 + m_2v_2^2 = m_2u_2^2$$

$$\Rightarrow \left(\frac{m_2^2}{m_1} + m_2\right)v_2^2 - 2\left(m_2u_1 + \frac{m_2^2}{m_1}u_2\right)v_2 + \left(2m_2u_1u_2 + \frac{m_2^2}{m_1}u_2^2 - m_2u_2^2\right)$$

$$\Rightarrow m_2\left(\frac{m_2 + m_1}{m_1}\right)v_2^2 - 2\left(m_2u_1 + \frac{m_2^2}{m_1}u_2\right)v_2 + \left(2m_2u_1u_2 + \frac{m_2^2}{m_1}u_2^2 - m_2u_2^2\right) \qquad \left|\frac{1}{m_2}\left(\frac{m_1}{m_2 + m_1}\right)\right.$$

$$\Rightarrow v_2^2 - 2\left(\frac{m_1}{m_2 + m_1}\right)\left(u_1 + \frac{m_2}{m_1}u_2\right)v_2 + \left(\frac{m_1}{m_2 + m_1}\right)\left(2u_1u_2 + \frac{m_2}{m_1}u_2^2 - u_2^2\right)$$

$$\Rightarrow v_2^2 - 2\left(\frac{1}{m_2 + m_1}\right)(m_1u_1 + m_2u_2)v_2 + \left(\frac{1}{m_2 + m_1}\right)\left(2m_1u_1u_2 + m_2u_2^2 - m_1u_2^2\right)$$

$$\Rightarrow v_{2_{1;2}} = \frac{m_1u_1 + m_2u_2}{m_2 + m_1} \pm \sqrt{\left(\frac{m_1u_1 + m_2u_2}{m_2 + m_1}\right)^2 - \left(\frac{2m_1u_1u_2 + m_2u_2^2 - m_1u_2^2}{m_2 + m_1}\right)}$$

Le calcul suivant nous démontre la complexité de la solution d'une équation quadratique

$$\Rightarrow v_{2_{1;2}} = \frac{m_1u_1 + m_2u_2}{m_2 + m_1} \pm$$

$$\sqrt{\left(\frac{m_1^2u_1^2 + 2m_1m_2u_1u_2 + m_2^2u_2^2}{(m_2 + m_1)^2}\right) - \left(\frac{2m_1m_2u_1u_2 + m_2^2u_2^2 - m_1m_2u_2^2}{(m_2 + m_1)^2}\right) - \left(\frac{2m_1^2u_1u_2 + m_2m_1u_2^2 - m_1^2u_2^2}{(m_2 + m_1)^2}\right)}$$

$$\Rightarrow v_{2_{1;2}} = \frac{m_1u_1 + m_2u_2}{m_2 + m_1} \pm$$

$$\sqrt{\left(\frac{m_1^2u_1^2 + 2m_1m_2u_1u_2 + m_2^2u_2^2 - 2m_1m_2u_1u_2 - m_2^2u_2^2 + m_1m_2u_2^2 - 2m_1^2u_1u_2 - m_2m_1u_2^2 + m_1^2u_2^2}{(m_2 + m_1)^2}\right)}$$

$$\Rightarrow v_{2_{1;2}} = \frac{m_1u_1 + m_2u_2}{m_2 + m_1} \pm \sqrt{\left(\frac{m_1^2u_1^2 - 2m_1^2u_1u_2 + m_1^2u_2^2}{(m_2 + m_1)^2}\right)} = \frac{m_1u_1 + m_2u_2}{m_2 + m_1} \pm \sqrt{\left(\frac{m_1^2\left(u_1^2 - 2u_1u_2 + u_2^2\right)}{(m_2 + m_1)^2}\right)}$$

$$\Rightarrow v_{2_{1;2}} = \frac{m_1u_1 + m_2u_2}{m_2 + m_1} \pm \sqrt{\frac{m_1^2(u_1 - u_2)^2}{(m_2 + m_1)^2}} = \frac{m_1u_1 + m_2u_2}{m_2 + m_1} \pm \frac{m_1(u_1 - u_2)}{(m_2 + m_1)}$$

$$\Rightarrow v_{2_1} = \frac{m_1u_1 + m_2u_2}{m_2 + m_1} + \frac{m_1(u_1 - u_2)}{(m_2 + m_1)} = \frac{m_1u_1 + m_2u_2 + m_1u_1 - m_1u_2}{m_2 + m_1} = \underline{\underline{\frac{2m_1u_1 + m_2u_2 - m_1u_2}{m_2 + m_1}}}$$

$$\Rightarrow v_{2_2} = \frac{m_1u_1 + m_2u_2}{m_2 + m_1} - \frac{m_1(u_1 - u_2)}{(m_2 + m_1)} = \frac{m_1u_1 + m_2u_2 - m_1u_1 + m_1u_2}{m_2 + m_1} = \underline{\underline{u_2}}$$

le calcul des vitesses du corps 1 avant et après la collision:

$$v_{1_1} = \frac{m_1u_1 + m_2(u_2 - v_2)}{m_1} \qquad \wedge \qquad v_{2_1} = \frac{2m_1u_1 + m_2u_2 - m_1u_2}{m_2 + m_1}$$

$$\Rightarrow v_{1_1} = \frac{m_1u_1 + m_2\left(u_2 - \frac{2m_1u_1 + m_2u_2 - m_1u_2}{m_2 + m_1}\right)}{m_1}$$

$$\Rightarrow v_{1_1} = \frac{m_1u_1 + m_2\left(\frac{(m_2 + m_1)u_2 - 2m_1u_1 - m_2u_2 + m_1u_2}{m_2 + m_1}\right)}{m_1}$$

$$\Rightarrow v_{1_1} = u_1 + \frac{m_2}{m_1}\left(\frac{m_2u_2 + m_1u_2 - 2m_1u_1 - m_2u_2 + m_1u_2}{m_2 + m_1}\right)$$

$$\Rightarrow v_{1_1} = \frac{m_2u_1 + m_1u_1 + m_2u_2 - 2m_2u_1 + m_2u_2}{m_2 + m_1} = \underline{\underline{\frac{m_1u_1 + 2m_2u_2 - m_2u_1}{m_2 + m_1}}}$$

===

$$v_{1_2} = \frac{m_1u_1 + m_2(u_2 - v_2)}{m_1} \qquad \wedge \qquad v_{2_2} = u_2$$

$$v_{1_2} = \frac{m_1u_1 + m_2(u_2 - u_2)}{m_1} = \underline{\underline{u_1}}$$

Solution numérique dans la situation initiale:

$$v_{1_1} = \frac{m_1u_1 + 2m_2u_2 - m_2u_1}{m_2 + m_1} \qquad v_{1_1} = \frac{0{,}6kg \cdot 4\,\frac{m}{s} + 2 \cdot 0{,}4kg \cdot 1\,\frac{m}{s} - 0{,}4kg \cdot 4\,\frac{m}{s}}{1kg}$$

$$\Rightarrow v_{1_1} = 1{,}6\,\frac{m}{s}$$

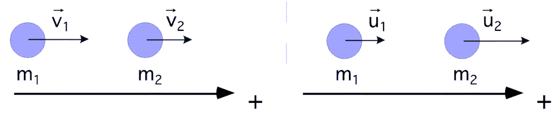
$$v_{2_1} = \frac{2m_1u_1 + m_2u_2 - m_1u_2}{m_2 + m_1} \qquad v_{2_1} = \frac{2 \cdot 0{,}6kg \cdot 4\,\frac{m}{s} + 0{,}4kg \cdot 1\,\frac{m}{s} - 0{,}6kg \cdot 1\,\frac{m}{s}}{1kg}$$

$$\Rightarrow v_{2_1} = 4{,}6\,\frac{m}{s}$$

22 donné: $m_1 = 2\,kg\,;\, m_2 = 1\,kg\,;\, v_2 = 2\,\frac{m}{s}\,;\, u_2 = 0\,\frac{m}{s}\,;\, el\,Stoss$

cherché: v_1

Solution: L'esquisse



Solution analytique

solution numérique

conservation de l'énergie et de la quantité de mouvement:

$$E: \quad \frac{m_1}{2}v_1^2 + \frac{m_2}{2}v_2^2 = \frac{m_1}{2}u_1^2$$

$$p: \quad m_1v_1 + m_2v_2 = m_1u_1 \qquad \Rightarrow u_1 = \frac{m_1v_1 + m_2v_2}{m_1}$$

$$\Rightarrow m_1v_1^2 + m_2v_2^2 = m_1\left(\frac{m_1v_1 + m_2v_2}{m_1}\right)^2$$

$$\Rightarrow m_1\left(\frac{m_1v_1 + m_2v_2}{m_1}\right)^2 = m_1v_1^2 + m_2v_2^2$$

$$\Rightarrow (m_1v_1 + m_2v_2)^2 = m_1^2v_1^2 + m_1m_2v_2^2$$

$$\Rightarrow m_1^2v_1^2 + 2m_1m_2v_1v_2 + m_2^2v_2^2 = m_1^2v_1^2 + m_1m_2v_2^2$$

$$\Rightarrow 2m_1m_2v_1v_2 = m_1m_2v_2^2 - m_2^2v_2^2 \qquad \left|\,\frac{1}{2m_1m_2v_2}\right.$$

$$\Rightarrow v_1 = \frac{m_2v_2^2(m_1 - m_2)}{2m_1m_2v_2} \qquad \wedge\, v_2 \neq 0$$

Solution numérique:

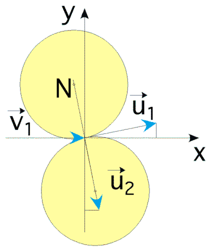
$$v_1 = \frac{m_2v_2^2(m_1 - m_2)}{2m_1m_2v_2} = \frac{1kg \cdot \left(2\,\frac{m}{s}\right)^2 (2kg - 1kg)}{2 \cdot 2kg \cdot 1kg \cdot \left(2\,\frac{m}{s}\right)} = 0{,}5\,\frac{m}{s}$$

$$u_1 = \frac{m_1v_1 + m_2v_2}{m_1} = \frac{2kg \cdot 0{,}5\,\frac{m}{s} + 1kg \cdot 2\,\frac{m}{s}}{2kg} = 1{,}5\,\frac{m}{s}$$

23 donné: $m_\alpha = m_{HE} = m_1 = m_2 = 6{,}6448 \cdot 10^{-27}\ kg$; $E_1 = 7{,}66 \cdot 10^{13}$; $v_2 = 0\ \frac{m}{s}$; $el\,Stoss$

cherché: v_1 ; v_2

Solution: L'esquisse



Solution analytique solution numérique
conservation de l'énergie et de la quantité de mouvement:

Solution numérique dans la situation finale:

$$E: \quad v_1^2 = u_1^2 + u_2^2 \qquad [1]$$

$$p: \quad \begin{pmatrix} v_1 \\ 0 \end{pmatrix} = \begin{pmatrix} u_{x1} \\ u_{y1} \end{pmatrix} + \begin{pmatrix} u_{x2} \\ u_{y2} \end{pmatrix} \qquad [2]$$

$$u_{x1} = u_1 \cos 30^0 = u_1 \tfrac{\sqrt{3}}{2}$$
$$u_{y1} = u_1 \sin 30^0 = u_1 \tfrac{1}{2}$$
$$u_{x2} = u_2 \sin 30^0 = u_2 \tfrac{1}{2}$$
$$u_{y2} = u_2 \cos 30^0 = u_2 \tfrac{\sqrt{3}}{2}$$

$$v_1 = u_{x1} + u_{x2} \quad \Rightarrow \quad v_1 = \tfrac{\sqrt{3}}{2} u_1 + \tfrac{1}{2} u_2 \qquad [2]$$

$$u_{y1} = u_{y2} \quad \Rightarrow \quad \tfrac{1}{2} u_1 = \tfrac{\sqrt{3}}{2} u_2 \quad \Rightarrow \quad \underline{u_1 = \sqrt{3}\ u_2} \qquad [3] \to [2]$$

$$\Rightarrow \quad v_1 = \sqrt{3}\ u_2 \tfrac{\sqrt{3}}{2} + \tfrac{1}{2} u_2 = 2 u_2 \quad \Rightarrow \quad \underline{\underline{u_2 = \tfrac{1}{2} v_1}}$$

$$\Rightarrow \quad \underline{\underline{u_1 = \tfrac{\sqrt{3}}{2} v_1}}$$

Solution numérique de la situation finale

$$E = m \tfrac{v_1^2}{2} \quad \Rightarrow \quad v_1 = \sqrt{\tfrac{2E}{m}}$$

$$\Rightarrow \quad \underline{\underline{u_2 = \tfrac{1}{2} v_1 = \tfrac{1}{2}\sqrt{\tfrac{E}{2m}} = 7{,}592 \cdot 10^6\ \tfrac{m}{s}}}$$

$$\Rightarrow \quad \underline{\underline{u_1 = \tfrac{\sqrt{3}}{2} v_1 = \tfrac{\sqrt{3}}{2}\sqrt{\tfrac{E}{2m}} = 13{,}150 \cdot 10^6\ \tfrac{m}{s}}}$$