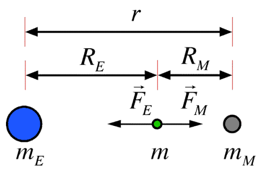
Lösungen: Gravitation

1 gegeben: $r = 3{,}9 \cdot 10^{8}\ m\,;\ m_M = 7{,}3 \cdot 10^{22}\ kg\,;\ m_E = 6 \cdot 10^{24}\ kg$

gesucht: R_E

Lösung: Skizze



analytische Lösung

$$F_E = F_M \Rightarrow G\frac{mM_E}{R_E^2} = G\frac{mM_M}{R_M^2} \Rightarrow \sqrt{\frac{M_E}{M_M}} = \frac{R_E}{R_M}$$

$$\Rightarrow R_E = (d - R_E)\sqrt{\frac{M_E}{M_M}}$$

numerische Lösung

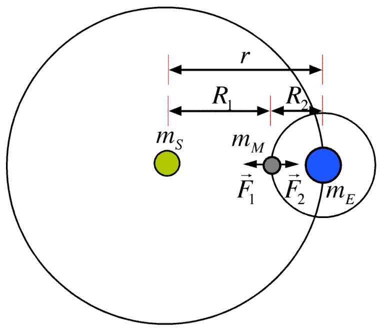
$$\Rightarrow \quad R_E = \frac{r\sqrt{\frac{M_E}{M_M}}}{1+\sqrt{\frac{M_E}{M_M}}} = \underline{\underline{3{,}5 \cdot 10^{8}\ m}}$$

2 gegeben:

$R_1 = 1{,}5 \cdot 10^{11}\ m\,;\ R_2 = 3{,}9 \cdot 10^{8}\ m\,;\ m_M = 7{,}3 \cdot 10^{22}\ kg\,;\ m_E = 6 \cdot 10^{24}\ kg\,;\ m_S = 2 \cdot 10^{30}\ kg$

gesucht: $\dfrac{F_1}{F_2}$

Lösung: Skizze



Analytische Lösung

$$\frac{F_1}{F_2} = \frac{G\frac{m_S m_M}{R_1^2}}{G\frac{m_E m_M}{R_2^2}} = \frac{m_S}{m_E}\frac{R_2^2}{R_1^2}$$

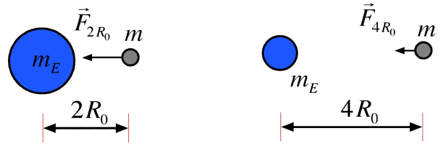
numerische Lösung

$$\underline{\underline{\frac{F_1}{F_2} = 2{,}25}}$$

3 gegeben: $r = 2 \cdot R_0 \qquad F_{G_{2 \cdot R_0}} = 144000\,N$

gesucht: $F_{G_{4 \cdot R_0}}$

Lösung: Skizze



Analytische Lösung

$$\frac{F_{G_{2R_0}}}{F_{G_{4R_0}}} = \frac{G\frac{m \cdot m_E}{(2R_0)^2}}{G\frac{m \cdot m_E}{(4R_0)^2}}$$

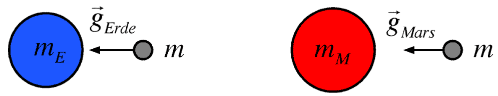
numerische Lösung

$$\frac{F_{G_{2R_0}}}{F_{G_{4R_0}}} = \frac{16}{4} = 4 \Rightarrow \qquad \underline{\underline{F_{G_{4R_0}} = 36'000\,N}}$$

4 gegeben: $G_{Terre} = 800\ N\,;\ g_{Mars} = 3{,}62\,\frac{m}{s^2}\,;\ g_{Erde} = 9{,}81\,\frac{m}{s^2}$

gesucht: G_{Mars}

Lösung: Skizze



Analytische Lösung

$$\left.\begin{aligned} G_{Erde} &= m \cdot g_{Erde} \\ G_{Mars} &= m \cdot g_{Mars} \end{aligned}\right\} \Rightarrow m = \frac{G_{Erde}}{g_{Erde}} = \frac{G_{Mars}}{g_{Mars}}$$

$$\Rightarrow G_{Mars} = \frac{G_{Erde}}{g_{Erde}} \cdot g_{Mars}$$

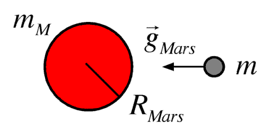
numerische Lösung

$$\underline{\underline{G_{Mars} = 295{,}2\,N}}$$

5 gegeben: $m_{Mars} = 6.42 \cdot 10^{23}\ kg\,;\ g_{Mars} = 3{,}62\,\frac{m}{s^2}\,;\ G = 6{,}673231 \cdot 10^{-11}\,\frac{N \cdot m^2}{kg^2}$

gesucht: R_{Mars}

Lösung: Skizze



Analytische Lösung

$$F = G\frac{m_{Mars} m}{R_{Mars}^2} \wedge g_{Mars} = G\frac{m_{Mars}}{R_{Mars}^2}$$

$$\Rightarrow R_{Mars} = \sqrt{\frac{G \cdot m_{Mars}}{g_{Mars}}}$$

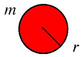
numerische Lösung

$$\underline{\underline{R_{Mars} = 3440\ km}}$$

6 gegeben: $g = \frac{1}{2} g_0$; $r = \frac{1}{2} R_0$; $m_E = 6 \cdot 10^{24}\ kg$

gesucht: m

Lösung: Skizze



Analytische Lösung

$$\left.\begin{array}{l} g = \dfrac{Gm}{r^2} \\ g_0 = \dfrac{Gm_E}{R_0^2} \end{array}\right\} \wedge g = \frac{1}{2} g_0 \Rightarrow \frac{Gm}{r^2} = \frac{1}{2} \frac{Gm_E}{R_0^2}$$

$$\Rightarrow m = \frac{1}{2} \frac{m_E}{R_0^2} r^2 \wedge r = \frac{1}{2} R_0 \qquad \Rightarrow m = \frac{1}{2} \frac{m_E}{R_0^2} \left(\frac{1}{2} R_0 \right)^2$$

$$\Rightarrow m = \frac{1}{8} m_E$$

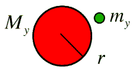
numerische Lösung

$$\underline{\underline{m = 7{,}5 \cdot 10^{23}\ kg}}$$

7 gegeben: $g_0 = 9{,}81 \frac{m}{s^2}$; $R_y = \frac{1}{2} R_0$; $M_y = \frac{1}{27} M_E$; $m = 70\ kg$; $M_E = 6 \cdot 10^{24}\ kg$

gesucht: G_y

Lösung: Skizze



Analytische Lösung

$$P_y = g_y m = G \frac{M_y}{R_y^2} m = G \frac{1}{27} M_E m \frac{9}{R_T^2} = \frac{1}{3} g_0 m$$

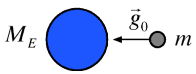
numerische Lösung

$$\underline{\underline{P_y = 228{,}9\ N}} \quad ; P_E = 686{,}7\ N$$

8 gegeben: $g_0 = 9{,}81 \frac{m}{s^2}$; $G = 6{,}673231 \cdot 10^{-11} N \frac{m^2}{kg^2}$; $R_0 = 6{,}38 \cdot 10^6 m$

gesucht: M_E

Lösung: Skizze



Analytische Lösung

$$g_0 = G \frac{M_E}{R_0^2} \qquad \Rightarrow M_E = \frac{g_0}{G} R_0^2$$

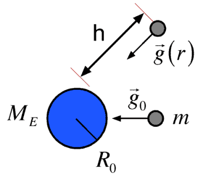
numerische Lösung

$$\underline{\underline{M_E = 5{,}98 \cdot 10^{24} kg}}$$

9 gegeben: $m = 50\ kg$; $h = 6380\ m$; $R_0 = 6380 \cdot 10^3 m$; $g_0 = 9{,}81 \frac{m}{s^2}$

gesucht: $a)\ G_0$; $b)\ \frac{G_h}{G_0}$

Lösung: Skizze



$$g(r) = G \frac{M_E}{r^2} = G \frac{M_E}{(R_0 + h)^2} = g_0 \frac{R_0^2}{(R_0 + h)^2}$$

Analytische Lösung

$a)\ G_0 = m \cdot g_0$

$$b)\ \frac{G_h}{G_0} = \frac{m g_0 \frac{R_0^2}{(R_0+h)^2}}{m g_0} = \frac{R_0^2}{(R_0 + h)^2}$$

numerische Lösung

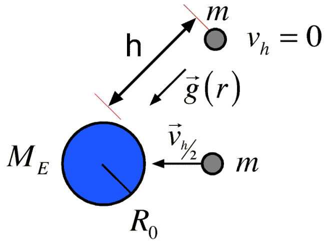
$a)\ \underline{\underline{G_0 = 490{,}5\ N}}$

$b)\ \underline{\underline{\frac{G_h}{G_0} = 0{,}998}}$

10 gegeben: $g_0 = 9{,}81\,\frac{m}{s^2}\,;\, G = 6{,}673231\cdot 10^{-11}\,N\,\frac{m^2}{kg^2}\,;\, R_0 = 6{,}38\cdot 10^{6}\,m\,;\, M_E = 6\cdot 10^{24}\,kg$

gesucht: $a)\ v_2 \quad ;b)\ h_{\frac{1}{2}v_2}$

Lösung: Skizze



Analytische Lösung numerische Lösung

$a) \quad \Delta E_{pot} + \Delta E_{kin} = 0$

$$\Rightarrow \left[-G\frac{M_E m}{R_0}\right] - \left[-G\frac{M_E m}{R_0 + h}\right] + \frac{m v_2^2}{2} = 0$$

$$\Rightarrow v_2 = \sqrt{\frac{2}{m}\left[\frac{GM_E m}{R_0} - \frac{GM_E m}{R_0 + h}\right]} = \sqrt{2GM_E\left[\frac{1}{R_0} - \frac{1}{R_0+h}\right]} \qquad \Rightarrow \underline{\underline{v_2 = 443\,\tfrac{m}{s}}}$$

$b) \quad \Delta E_{pot} + \Delta E_{Kin} = 0$

$$\Rightarrow \left[-G\frac{M_E m}{R_0 + h_{\frac{1}{2}v}}\right] - \left[-G\frac{M_E m}{R_0 + h}\right] + \frac{m\left(\frac{v_2}{2}\right)^2}{2} - \frac{m v_h^2}{2} = 0 \qquad \wedge \qquad v_h = 0$$

$$\Rightarrow \frac{1}{R_0 + h_{\frac{1}{2}v}} - \frac{1}{R_0 + h} = \frac{v_2^2}{8GM_E} \qquad \Rightarrow \frac{1}{R_0 + h_{\frac{1}{2}v}} = \frac{1}{R_0 + h} + \frac{v_2^2}{8GM_E}$$

$$\Rightarrow \frac{1}{R_0 + h_{\frac{1}{2}v}} = \frac{8GM_E + v_2^2(R_0 + h)}{(R_0 + h)8GM_E}$$

$$\Rightarrow \frac{1}{R_0 + h_{\frac{1}{2}v}} = \frac{8GM_E + 2GM_E\left[\frac{1}{R_0} - \frac{1}{R_0 + h}\right](R_0 + h)}{(R_0 + h)8GM_E} = \frac{8 + 2\left(\frac{R_0 + h - R_0}{R_0}\right)}{(R_0 + h)8}$$

$$\Rightarrow h_{\frac{1}{2}v} = \frac{(R_0 + h)}{R_0 + \frac{1}{4}h}R_0 - R_0 = R_0\left(\frac{R_0 + h}{R_0 + \frac{1}{4}h} - 1\right) = R_0\left(\frac{4(R_0 + h) - (4R_0 + h)}{4R_0 + h}\right)$$

$$\Rightarrow h_{\frac{1}{2}v} = \frac{3h}{4R_0 + h}R_0 \qquad \Rightarrow \underline{\underline{h_{\frac{1}{2}v} = 2{,}5\,km}}$$

11 gegeben:

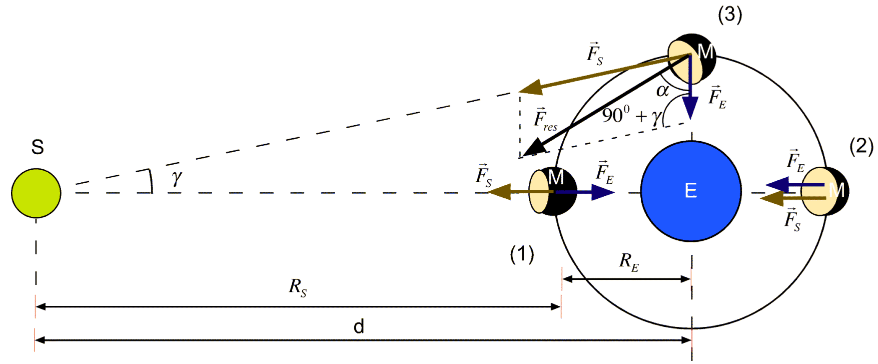
$d = 1{,}496\cdot 10^{11}\,m \qquad M_S = 1{,}989\cdot 10^{30}\,kg \qquad G = 6{,}6732\cdot 10^{11}\,\frac{Nm^2}{kg^2}$

$R_E = 3{,}844\cdot 10^{8}\,m \qquad M_E = 5{,}976\cdot 10^{24}\,kg$

$M_M = 7{,}35\cdot 10^{22}\,kg$

gesucht: $a)\ F_{res_i}\ , i = 1,2,3 \qquad b)\ Begründung\ Situation\ (3)$

Lösung: Skizze



Analytische Lösung numerische Lösung

Berechnung bei NEUMOND (1)

$$F_{res} = F_S - F_E = GM_M\left(\frac{M_S}{R_S^2} - \frac{M_E}{R_E^2}\right) = \underline{\underline{2{,}398\cdot 10^{20}\,N}}$$

Berechnung bei Vollmond (2)

$$F_{res} = F_S + F_E = GM_M\left(\frac{M_S}{R_S^2} + \frac{M_E}{R_E^2}\right) = \underline{\underline{6{,}330\cdot 10^{20}\,N}}$$

Berechnung bei Halbmond (3)

$$F_{res} = \sqrt{F_E^2 + F_S^2 - 2F_E F_S\cos\left(90^0 + \gamma\right)}$$

$$\tan\gamma = \frac{R_E}{d} \qquad \Rightarrow \gamma = \arctan\frac{R_E}{d}$$

$$\frac{\sin\alpha}{\sin\left(90^0 + \gamma\right)} = \frac{F_S}{F_{res}} \qquad \Rightarrow \alpha = \arcsin\left[\sin\left(90^0 + \gamma\right)\cdot\tfrac{F_S}{F_{res}}\right]$$

$$\underline{\underline{F_{res} = 4{,}794\cdot 10^{20}\,N}}\,;\,\underline{\underline{\alpha = 65{,}4096^0}}$$

b) Betrachtet man die gesamte Bewegung des Mondes während eines Jahres, dann bemerkt man, dass diese Bewegung sehr kompliziert ist und aus der Überlagerung von zwei annährend exakten Kreisbewegungen zusammengesetzt ist.

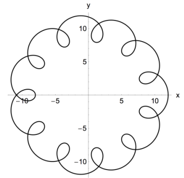
Die Mondbahn um die Sonne während einem Jahr

$\alpha_T = \omega_T \cdot t$

$\alpha_L = \omega_L \cdot t$

$x(t) = R_T \cos(\omega_T \cdot t) + R_L \cos(\omega_L \cdot t)$

$y(t) = R_T \sin(\omega_T \cdot t) + R_L \sin(\omega_L \cdot t)$



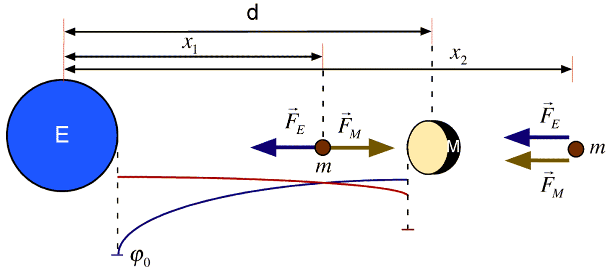
12 gegeben:

$d_{SE} = 1{,}496 \cdot 10^{11}\ m\,;\ d_{EM} = 3{,}844 \cdot 10^{8}\ m\,;\ R_E = 6{,}371 \cdot 10^{6}\ m$

$M_S = 1{,}989 \cdot 10^{30}\ kg\,;\ M_E = 5{,}976 \cdot 10^{24}\ kg\,;\ M_M = 7{,}35 \cdot 10^{22}\ kg\,;\ G = 6{,}6732 \cdot 10^{-11}\ \frac{Nm^2}{kg^2}$

gesucht: $F_M(x) = F_E(x) \Rightarrow a)\ \varphi_{res}(x) \quad b)\ \frac{\varphi_{res}}{\varphi_0}\ c)\ x$

Lösung: Skizze



Analytische Lösung numerische Lösung

$$a) \quad F_M(x) = F_E(x) \Rightarrow G\frac{M_M m}{(d_{ME} - x)^2} = G\frac{M_E m}{x^2} \qquad \wedge\ d := d_{ME}$$

$$\Rightarrow M_M x^2 = M_E (d-x)^2 \qquad \Rightarrow x^2 = \frac{M_E}{M_M}(d-x)^2 \qquad \wedge\ c := \frac{M_E}{M_M}$$

$$\Rightarrow x^2 = c \cdot d^2 - 2dcx + cx^2 \qquad \Rightarrow x^2(1-c) + 2c \cdot d \cdot x - cd^2 = 0$$

$$\Rightarrow x^2 + \frac{2c \cdot d}{(1-c)} \cdot x - \frac{cd^2}{(1-c)} = 0 \qquad \Rightarrow x_{1;2} = -\frac{c \cdot d}{(1-c)} \pm \sqrt{\frac{c^2 \cdot d^2 + cd^2(1-c)}{(1-c)^2}}$$

$$\underline{\underline{x_{1;2} = -\frac{c \cdot d}{(1-c)} \pm \sqrt{\frac{cd^2}{(1-c)^2}}}}$$

$\Rightarrow \underline{\underline{x_1 = 3{,}46 \cdot 10^8\, m}}$ (zwischen Mond und Erde)

$\wedge \quad \vec{F}_M = -\vec{F}_E \wedge F_M = F_E$

$\Rightarrow \underline{\underline{x_2 = 4{,}32 \cdot 10^8\, m}}$ (ausserhalb der Verbindungslinie Erde – Mond)

$\wedge \quad \vec{F}_M = \vec{F}_E \wedge F_M = F_E$

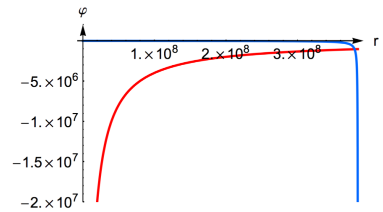
$$\varphi_{res} = \varphi_E + \varphi_M \qquad \varphi_E = -G\frac{M_E}{x_1} \wedge \qquad \varphi_M = -G\frac{M_M}{d - x_1}$$

$$\Rightarrow \underline{\underline{\varphi_{res}(x_1) = -1.280 \cdot 10^6\ \tfrac{J}{kg}}} \qquad \Rightarrow \underline{\underline{\varphi_E(x_1) = -1.152 \cdot 10^6\ \tfrac{J}{kg}}} \qquad \Rightarrow \underline{\underline{\varphi_M(x_1) = -0{,}1278 \cdot 10^6\ \tfrac{J}{kg}}}$$

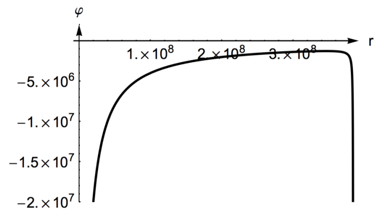
$$\Rightarrow \underline{\underline{\frac{\varphi_{res}}{\varphi_0} = 0{,}02}}$$

b) Maximum der resultierenden Potentialkurve

Das Potential der Erde und des Mondes



Das resultierende Potential

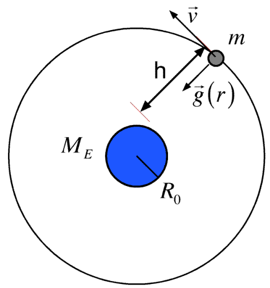


Der neutrale Punkt der Gravitationskräfte befindet sich $54\ R_0$ $(R_0 = \textit{Radius der Erde})$ vom Zentrum der Erde und etwa $6\ R_0$ vom Zentrum des Mondes.

13 gegeben: $G\,;\ M_E\,;\ R_0\,;\ m\,;\ r$

gesucht: E_{tot}^{sat}

Lösung: Skizze



$$\varphi(r) = -G\frac{M_E}{r}$$

Analytische Lösung numerische Lösung

$$E_{tot}^{sat} = GM_E m\left(\frac{1}{R_0} - \frac{1}{2r}\right)$$

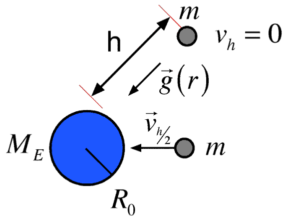
14 gegeben:

$g_0 = 9{,}81\,\frac{m}{s^2}\,;\, G = 6{,}673231\cdot 10^{-11}\,N\,\frac{m^2}{kg^2}\,;\, R_0 = 6{,}38\cdot 10^6\,m\,;\, M_E = 6\cdot 10^{24}\,kg$

$h = 10000\,km = 10^7\,m$

gesucht: $a)\, v_2 \quad ;b)\, h_{\frac{1}{2}v_2}$

Lösung: Skizze



Analytische Lösung numerische Lösung

$$a)\qquad \Delta E_{pot} + \Delta E_{kin} = 0$$

$$\Rightarrow \left[-G\frac{M_E m}{R_0}\right] - \left[-G\frac{M_E m}{R_0 + h}\right] + \frac{m v_2^2}{2} = 0$$

$$\Rightarrow v_2 = \sqrt{\frac{2}{m}\left[\frac{G M_E m}{R_0} - \frac{G M_E m}{R_0 + h}\right]} = \sqrt{2 G M_E \left[\frac{1}{R_0} - \frac{1}{R_0 + h}\right]} \qquad \Rightarrow \underline{\underline{v_2 = 8734\,\tfrac{m}{s}}}$$

$$b)\qquad \Delta E_{pot} + \Delta E_{kin} = 0$$

$$\Rightarrow \left[-G\frac{M_E m}{R_0 + h_{\frac{1}{2}v}}\right] - \left[-G\frac{M_E m}{R_0 + h}\right] + \frac{m\left(\frac{v_2}{2}\right)^2}{2} - \frac{m v_h^2}{2} = 0 \qquad \wedge \qquad v_h = 0$$

$$\Rightarrow \frac{1}{R_0 + h_{\frac{1}{2}v}} - \frac{1}{R_0 + h} = \frac{v_2^2}{8 G M_E} \qquad \Rightarrow \frac{1}{R_0 + h_{\frac{1}{2}v}} = \frac{1}{R_0 + h} + \frac{v_2^2}{8 G M_E}$$

$$\Rightarrow \frac{1}{R_0 + h_{\frac{1}{2}v}} = \frac{8 G M_E + v_2^2 (R_0 + h)}{(R_0 + h) 8 G M_E}$$

$$\Rightarrow \frac{1}{R_0 + h_{\frac{1}{2}v}} = \frac{8 G M_E + 2 G M_E \left[\frac{1}{R_0} - \frac{1}{R_0 + h}\right](R_0 + h)}{(R_0 + h) 8 G M_E} = \frac{8 + 2\left(\frac{R_0 + h - R_0}{R_0}\right)}{(R_0 + h) 8}$$

$$\Rightarrow h_{\frac{1}{2}v} = \frac{(R_0 + h)}{R_0 + \frac{1}{4}h} R_0 - R_0 = R_0\left(\frac{R_0 + h}{R_0 + \frac{1}{4}h} - 1\right) = R_0\left(\frac{4(R_0 + h) - (4R_0 + h)}{4R_0 + h}\right)$$

$$\Rightarrow h_{\frac{1}{2}v} = \frac{3h}{4R_0 + h} R_0 \qquad \Rightarrow \underline{\underline{h_{\frac{1}{2}v} = 5386\,km}}$$

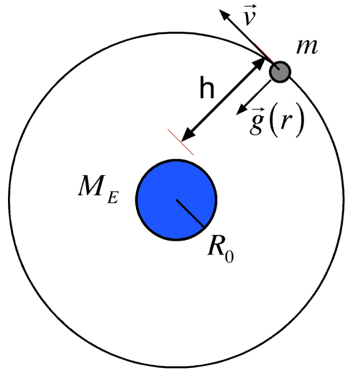
15 gegeben:

$g_0 = 9{,}81\,\frac{m}{s^2}\,;\, G = 6{,}673231\cdot 10^{-11}\,N\,\frac{m^2}{kg^2}\,;\, R_0 = 6{,}38\cdot 10^6\,m\,;\, M_E = 6\cdot 10^{24}\,kg$

$h = 10000\,km = 10^7\,m$

gesucht: $a)\, v;\quad b)\, T\,;\quad c)\, \frac{E}{m}$

Lösung: Skizze



Analytische Lösung numerische Lösung

$$a)\, F_G = F_Z \qquad \Rightarrow \qquad G\frac{M_E m}{(R_0 + h)^2} = m r \omega^2 = m\frac{v^2}{r}$$

$$\Rightarrow v = \sqrt{\frac{G M_E}{R_0 + h}} \qquad \underline{\underline{v = 7{,}35\cdot 10^3\,\tfrac{m}{3}}}$$

$$b)\, v = \tfrac{2\pi(R_0 + h)}{T} \qquad \Rightarrow T = \tfrac{2\pi(R_0 + h)}{v} \qquad \underline{\underline{T = 6301\,s}}$$

$$c)\, \tfrac{E}{m} = \varphi(R_0 + h) - \varphi_0 + \tfrac{v^2}{2} \qquad \underline{\underline{\tfrac{E}{m} = 3{,}55\cdot 10^7\,\tfrac{J}{kg}}}$$

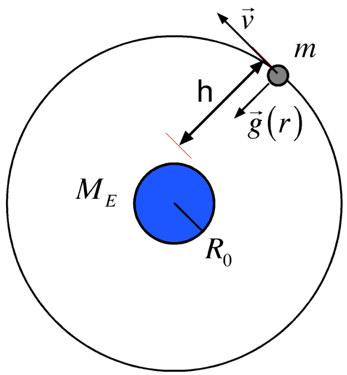
16 gegeben:

$g_0 = 9{,}81\,\frac{m}{s^2}\,;\; G = 6{,}673231\cdot 10^{-11}\,N\,\frac{m^2}{kg^2}\,;\; R_0 = 6{,}38\cdot 10^6\,m\,;\; M_E = 6\cdot 10^{24}\;kg$

$a)\, h = 0\,m\,;\; b)\, h = R_0\,;\; c)\, h \to \infty$

gesucht: $a)\, v_{h=0}\,;\left(\frac{E}{m}\right)_{h=0}\,;\; b)\, v_{h=R_0}\,;\left(\frac{E}{m}\right)_{h=R_0}\,;\; c)\, v_{Flucht}\,;\left(\frac{E}{m}\right)_{Flucht}$

Lösung: Skizze



Analytische Lösung | numerische Lösung

$a)\, F_G = F_Z \quad\Rightarrow\quad mg_0 = mr\omega^2 = m\frac{v^2}{R_0}$

$\Rightarrow v_{h=0} = \sqrt{g_0 R_0}$ | $\underline{\underline{v_{h=0} = 7906\,\frac{m}{3}}}$

$\left(\frac{E}{m}\right)_{h=0} = \frac{v^2}{2}$ | $\underline{\underline{\left(\frac{E}{m}\right)_{h=0} = 3{,}125\cdot 10^7\,\frac{J}{kg}}}$

$b)\, F_G = F_Z \quad\Rightarrow\quad G\frac{M_E m}{4R_0^2} = mr\omega^2 = m\frac{v^2}{2R_0}$

$v_{h=R_0} = \sqrt{\frac{GM_E}{2R_0}}$ | $\underline{\underline{v_{h=R_0} = 5594\,\frac{m}{3}}}$

$\left(\frac{E}{m}\right)_{h=R_0} = \varphi(2R_0) - \varphi_0 + \frac{v_{h=R_0}^2}{2}$ | $\underline{\underline{\left(\frac{E}{m}\right)_{h=R_0} = 4{,}69\cdot 10^7\,\frac{J}{kg}}}$

$c)\, v_{Flucht} = \sqrt{-2\varphi_0}$ | $\underline{\underline{v_{Flucht} = 11'200\,\frac{m}{s}}}$

$\left(\frac{E}{m}\right)_{Flucht} = \frac{v_{Flucht}^2}{2} = \varphi_0$ | $\underline{\underline{\left(\frac{E}{m}\right)_{Flucht} = 6{,}27\cdot 10^7\,\frac{J}{kg}}}$

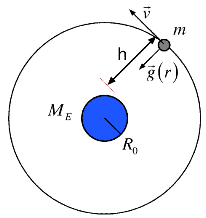
17 gegeben:

$g_0 = 9{,}81\,\frac{m}{s^2}\,;\; G = 6{,}673231\cdot 10^{-11}\,N\,\frac{m^2}{kg^2}\,;\; R_0 = 6{,}38\cdot 10^6\,m\,;\; M_E = 6\cdot 10^{24}\;kg$

h

gesucht: $a)\, E_{pot}(h) \qquad ;\, b)\, E_{kin}(h)$

Lösung: Skizze



Analytische Lösung | numerische Lösung

$a)\, F_G = F_C \quad\Rightarrow\quad G\frac{M_T m}{(R_0+h)^2} = mr\omega^2 = m\frac{v^2}{R_0+h}$

$v_h = \sqrt{\frac{GM_T}{R_0+h}}$ | $\underline{\underline{E_{cin}(h) = \frac{m}{2}\cdot\frac{GM_T}{R_0+h}}}$

$b)$ | $\underline{\underline{E_{pot}(h) = -\frac{GM_T m}{R_0+h}}}$

$\underline{\underline{\frac{E_{cin}(h)}{E_{pot}(h)} = -\frac{1}{2}}}$

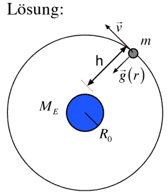
18 gegeben:

$g_0 = 9{,}81\,\frac{m}{s^2}\,;\; G = 6{,}673231\cdot 10^{-11}\,N\,\frac{m^2}{kg^2}\,;\; R_0 = 6{,}38\cdot 10^6\,m\,;\; M_E = 6\cdot 10^{24}\;kg$

$m = 800\,kg\,;\; h_1 = 300\,km\,;\; h_2 = 250\,km$

gesucht: $a)\, v_1\,;\, E_{kin_1}\,;\, E_{pot_1}\,;\, b)\, v_2\,;\, E_{kin_2}\,;\, E_{pot_2}\,;\, c)\, \Delta E_{pot}\,;\, \Delta E_{kin}$

Lösung: Skizze



Analytische Lösung | numerische Lösung

$F_G = F_Z \quad\Rightarrow\quad G\frac{M_E m}{(R_0+h)^2} = mr\omega^2 = m\frac{v^2}{R_0+h}$

$v_h = \sqrt{\frac{GM_E}{R_0+h}}$

$a)\quad \underline{\underline{v_1 = 7728\,\frac{m}{s}}}\,;\; \underline{\underline{E_{kin_1} = 2{,}389\cdot 10^{10}\,J}}\,;\, \underline{\underline{E_{pot_1} = 2{,}247\cdot 10^9\,J}}$

$b)\quad \underline{\underline{v_2 = 7757\,\frac{m}{s}}}\,;\; \underline{\underline{E_{kin_2} = 2{,}407\cdot 10^{10}\,J}}\,;\, \underline{\underline{E_{pot_2} = 1{,}887\cdot 10^9\,J}}$

$c)\quad \underline{\underline{\Delta v = 29\,\frac{m}{s}}} \quad \Rightarrow \underline{\underline{\Delta E_{kin} = 1{,}8\cdot 10^8\,J}}\,;\, \underline{\underline{\Delta E_{pot} = -3{,}6\cdot 10^8\,J}} \quad \Rightarrow \underline{\underline{\Delta E_{pot} = -2\cdot\Delta E_{kin}}}$