Beweis des Satzes von Heron

Der Satz des Heron

$$\boxed{F_\Delta = \sqrt{s(s-a)(s-b)(s-c)}} \qquad \text{mit} \qquad \boxed{s = \frac{a+b+c}{2}}$$

Beweis

$$F_\Delta = \sqrt{\left[s(s-c)\right]\left[(s-a)(s-b)\right]}$$

$$F_\Delta = \sqrt{\left[\frac{(a+b+c)}{2}\frac{(a+b-c)}{2}\right]\left[\frac{(b+c-a)}{2}\frac{(a+c-b)}{2}\right]}$$

$$F_\Delta = \sqrt{\left[\frac{(a+b+c)}{2}\frac{(a+b-c)}{2}\right]\left[\frac{(c-(a-b))}{2}\frac{(c+(a-b))}{2}\right]}$$

$$F_\Delta = \sqrt{\left[\frac{(a+b)^2-c^2}{4}\right]\left[\frac{c^2-(a-b)^2}{4}\right]}$$

$$F_\Delta = \sqrt{\left[\frac{(a+b)^2-(\vec{a}-\vec{b})^2}{4}\right]\left[\frac{(\vec{a}-\vec{b})^2-(a-b)^2}{4}\right]}$$

$$F_\Delta = \sqrt{\left[\frac{ab+\vec{a}\cdot\vec{b}}{2}\right]\left[\frac{ab-\vec{a}\cdot\vec{b}}{2}\right]}$$

$$F_\Delta = \frac{1}{2}\sqrt{(ab+\vec{a}\cdot\vec{b})(ab-\vec{a}\cdot\vec{b})}$$

$$F_\Delta = \frac{1}{2}\sqrt{(ab)^2-(\vec{a}\cdot\vec{b})^2}$$

$$F_\Delta = \frac{1}{2}\sqrt{(ab)^2-(ab)^2\cos^2\alpha}$$

$$F_\Delta = \frac{1}{2}ab\cdot\sqrt{1-\cos^2\alpha}$$

$$\boxed{F_\Delta = \frac{1}{2}ab\cdot\sin\alpha = \frac{1}{2}\left|\vec{a}\times\vec{b}\right|}$$