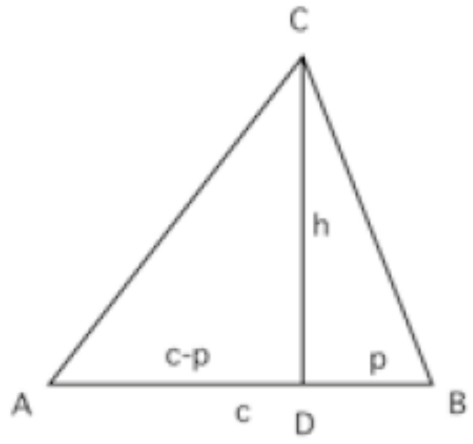
Beweis Satz des Heron



$$A=\sqrt{s(s-a)(s-b)(s-c)}$$
$$s=\frac{a+b+c}{2}$$

s: Halber Umfang des Dreiecks

Beweis:

$$A=\frac{c\cdot h}{2} \qquad [1]$$

$$h^2=a^2-p^2=(a+p)(a-p) \qquad [2]$$

$$h^2=b^2-(c-p)^2 \qquad [3]$$

$$[2]=[3] \qquad \Rightarrow a^2-p^2=b^2-(c-p)^2$$

$$\Rightarrow a^2-p^2=b^2-c^2+2pc-p^2$$

$$\Rightarrow p=\frac{a^2-b^2+c^2}{2c} \qquad \to [2]$$

$$\Rightarrow h^2=(a+p)(a-p)=\left(a+\frac{a^2-b^2+c^2}{2c}\right)\left(a-\frac{a^2-b^2+c^2}{2c}\right)$$

$$\Rightarrow h^2=\frac{2ac+a^2-b^2+c^2}{2c}\cdot\frac{2ac-a^2+b^2-c^2}{2c}$$

$$\Rightarrow h^2=\frac{(a+c)^2-b^2}{2c}\cdot\frac{b^2-(a-c)^2}{2c}$$

$$\Rightarrow h^2=\frac{\left[(a+c)^2-b^2\right]\cdot\left[b^2-(a-c)^2\right]}{4c^2}$$

$$\Rightarrow h^2=\frac{\left[(a+b+c)(a-b+c)\right]\cdot\left[(b-a+c)(b+a-c)\right]}{4c^2}$$

$$\wedge \qquad 2s:=a+b+c \qquad [4]$$

$$\Rightarrow h^2=\frac{\left[(2s)(2s-2b)\right]\cdot\left[(2s-2a)(2s-2c)\right]}{4c^2}$$

$$\Rightarrow h^2=\frac{16}{4c^2}s(s-a)(s-b)(s-c)=\frac{4}{c^2}s(s-a)(s-b)(s-c)$$

$$\Rightarrow h=\frac{2}{c}\sqrt{s(s-a)(s-b)(s-c)}$$

$$\Rightarrow A=\frac{c\cdot h}{2}=\sqrt{s(s-a)(s-b)(s-c)}$$

$$\wedge \qquad [4]\Rightarrow s=\frac{a+b+c}{2} \qquad q.e.d$$