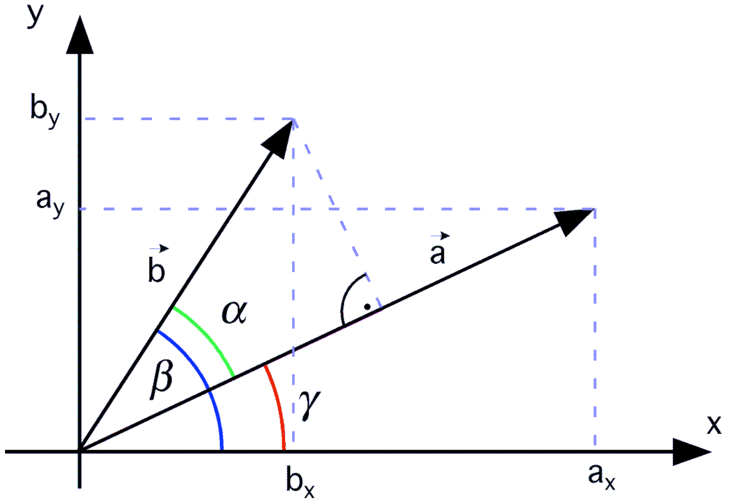
Beweis der Gleichheit der Definitionen des Skalarproduktes

Skizze



$$\textit{Definition}: \quad \alpha = \measuredangle(\vec{a};\vec{b})$$

$$\textit{zu zeigen}: \quad \vec{a}\cdot\vec{b} \overset{2)}{=} |\vec{a}|\cdot|\vec{b}|\cos\alpha \overset{1)}{=} a_x b_x + a_y b_y$$

Beweis 1):

$$\vec{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} = a_x\vec{i} + a_y\vec{j} \quad ; \qquad \vec{b} = \begin{pmatrix} b_x \\ b_y \end{pmatrix} = b_x\vec{i} + b_y\vec{j}$$

$$\Rightarrow \underline{\underline{\vec{a}\cdot\vec{b} = \left(a_x\vec{i} + a_y\vec{j}\right)\cdot\left(b_x\vec{i} + b_y\vec{j}\right) = a_x b_x + a_y b_y}}$$

Beweis 2)

$$\vec{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} a\cos\gamma \\ a\sin\gamma \end{pmatrix} \quad ; \qquad \vec{b} = \begin{pmatrix} b_x \\ b_y \end{pmatrix} = \begin{pmatrix} b\cos\beta \\ b\sin\beta \end{pmatrix}$$

$$\Rightarrow \vec{a}\cdot\vec{b} = \begin{pmatrix} a\cos\gamma \\ a\sin\gamma \end{pmatrix}\cdot\begin{pmatrix} b\cos\beta \\ b\sin\beta \end{pmatrix} = ab\left(\cos\gamma\cos\beta + \sin\gamma\sin\beta\right)$$

$$\Rightarrow \underline{\underline{\vec{a}\cdot\vec{b} = ab\cos\left(\gamma - \beta\right) = ab\cos\alpha}}$$